Here are **MY SOLUTIONS** to my final. I emphasize **MY** because there may be other valid approaches.

DIAGRAMS

Original diagrams are in black and boxed in.

Non-black items in these diagrams are add-ins for the solution.

All other diagrams are added for the solution

TEXT

Original text is black

MY SOLUTIONS are blue.

Statements not needed as part of the solution are in purple.

INSTRUCTIONS DO Problems 1,2 and 3, then any 5 of the remaining 9 problems. Clearly indicate those problems that you want graded by crossing out any work that you don't want graded . Otherwise I grade the first five (5) that have any work.

NAME ______Me____ ________My Solutions______________________Hulan E. Jack Jr.____

Borough of Manhattan Community College — Course ***Physics 215***

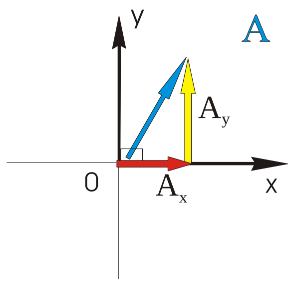
Instructor: ***Dr. Hulan E. Jack Jr.*** — Date **December 18, 2003**

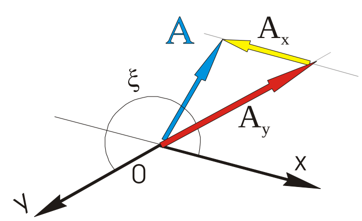
Final Exam- My Solutions

1. [12 pts Total]

a. What is a vector? Describe its defining features. [4 pts]

It is a quantitative entity describe by both magnitude and direction.





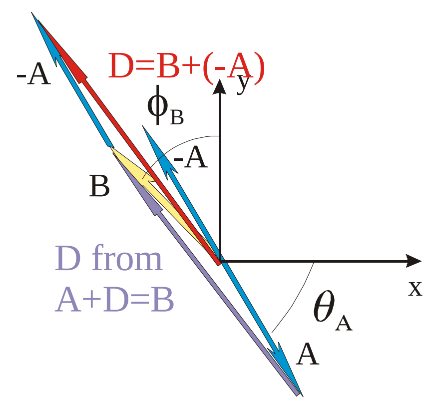
b. The vector A is shown in two coordinates systems. On each coordinate system sketch the x and y components of A for that system. **Briefly** explain the reason for your solutions. **[8 pts]** (4 pts each)

The components of a vector are a set of vectors, one parallel to each axis , whose sum is the original vector. This holds for each of the above situations.

2. **[16 pts Total]** The figure shows two vectors **A** and **B** .

A = 15 units, **B**=10 units and $\theta_A = 60^o$ and $\phi_B = 45^o$.

a. Sketch the vector **D** = **B** -**A** *directly on the figure*. [3 pts]



b. Write the components of the two vectors [4 pts]

Symbols	**Values**	**Symbols**	**Values**
A_x =	$A\cos\theta_A = 15\cos60^o$	B_x	$= -B\sin\phi_B = -10\sin45^o$
A_y =	$-A\sin\theta_A = -15\sin60^o$	B_y	$= B\cos\phi_B = 10\cos45^o$

c. Write the expressions for the components, magnitude D and direction, θ_D , of the vector **D = B - A** in terms of A_x, A_y, B_x , and B_y.

[9 pts total] **SYMBOLS ONLY**

$D_x = B_x - A_x$ $D_y = B_y - A_y$

$D = \text{sqrt}(D_x^2 + D_y^2) = \text{sqrt}([B_x - A_x]^2 + [B_y - A_y]^2)$

$\theta_D = \arctan (D_y/D_x) = \arctan ((B_y - A_y)/(B_x - A_x))$.

3. **[12 pts Total]** Using only the following information 1 Cal = 1000cal , 1 cal = 4.18J, 1 watt = 1J/s,
1 day = 24 hr, 1 hr = 60 min, 1 min = 60 s.

a. Write the pseudo code for Cal -> J and day -> s [2 pts]

Cal -> cal -> J , and day -> hr -> min -> s .

b. Set up the program to go from 1000 Cal/day to watt (1000 Cal/day = (?) watts) using **all** of the above information (**no numbers yet - just units**) [4 pts]

$$1000\frac{Cal}{day} = 1000\frac{Cal(1000)\frac{cal}{Cal}(4.18)\frac{J}{cal}}{day(24)\frac{hr}{day}(60)\frac{\min}{hr}(60)\frac{s}{\min}}$$

c. Fill in the numbers **in the above setup program**. [2 pts]
(OK, now numbers)

d. Explain, with an illustration, how to check the numbers for correct positions. [4 pts]

On substituting Cal =1000cal, if the 1000's cancel, then position is OK. If they don't, as on the right, invert to correct it.

$$\left(\frac{1}{1000}\right)\frac{\text{cal}}{\text{Cal}=1000}$$ do not cancel

$$\left(\frac{1000}{1}\right)\frac{\text{cal}}{\text{Cal}=1000}$$ corrected!!

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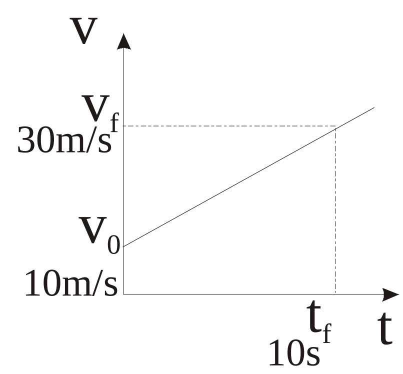
4. **[12 pts Total]** The velocity vs time (v vs t) graph of a body is as shown



a. The displacement over the time t=0 to t_f is what of this graph. [3pts]

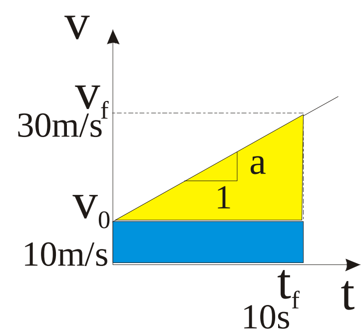
The displacement x is the area under the v vs t curve.

b. Obtain the symbolic expression for x in terms of v_f , v_0 and t_f . [3pts]

x = area under v vs t curve
= triangle + rectangle
= ½ base x height + base x height
= ½ t_f $(v_f - v_0)$ + t_f v_0 = ½ t_f (v_f+v_0)

c. The numerical value in m (meters) [2pts]

= ½ 10s x 40m/s = 200 m



d. Give the definition of acceleration and describe what it is on this graph [4 pts]

The acceleration a is the slope of the v vs t curve.
$a = \Delta v/\Delta t = (v_f - v_0)/t_f$

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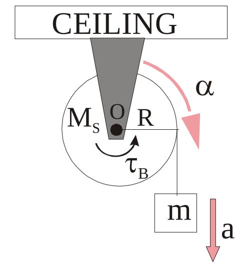
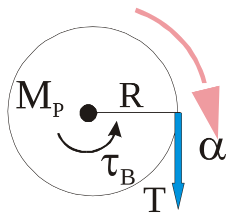
The next two problems, 5 and 6, involve a spool of cord with a mass m on the end of the cord, as shown. The spool has a radius R, mass M_S and moment of inertia about its center I_O . The spool also has a frictional torque τ_B in the bearing. As the cord unwinds the spool rotates without slipping.



5. **[12 pts Total]** Find the acceleration, a, of the mass m. Note that $\alpha = a/R$.

Sketch the FBD of the spool. [2pts] — State the physical Principles [2pts] — Fill in the Details [2pts]

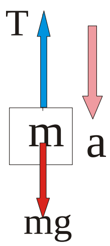


+cw $\Sigma \tau_O = I_O \alpha$, $\quad TR - \tau_B = I_O \alpha$,

Divide through by R $\quad T - \tau_B = I_O/R\ \alpha$, (1)

Use $\alpha = a/R$ $\quad T - \tau_B/R = (I_O/R^2)a$ (1a)

Sketch the FBD of the m. [2pts] — State the physical Principles [2pts] — Fill in the Details [2pts]



+down $\Sigma F = ma$, $\quad mg - T = ma$ (2)

Adding (1a) and (2), and a little algebra gives

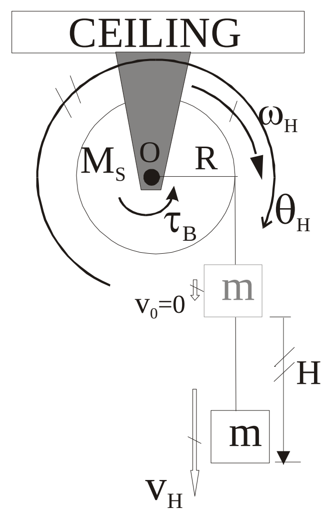
$$a = \frac{mg - \tau_B/R}{m + I_O/R^2}$$

and from (2)

$$T = m(g - a)$$

$$= m\left(g - \frac{mg - \tau_B/R}{m + I_O/R^2}\right)$$

6. **[12 pts Total]** Starting from rest , the mass, m, falls a displacement H, while the spool undergoes an angular displacement θ_H . Find the velocity v_H of the mass m after it has fallen by H.



a. State the Physical Principle (the formula) . [2 pts]

$E_{P\ init} + E_{K\ init} = E_{P\ final} + E_{K\ final} + Work_{Dissipp\ A\to B}$

b. Fill in the details. [8 pts]

$mgH + 0 = 0 + ½ mv_H^2 + ½ I_O \omega_H^2 + \tau_B\theta_H$.

$mgH + 0 = 0 + ½ mv_H^2 + ½ (I_O/R^2) v_H^2 + (\tau_B/R)H$.

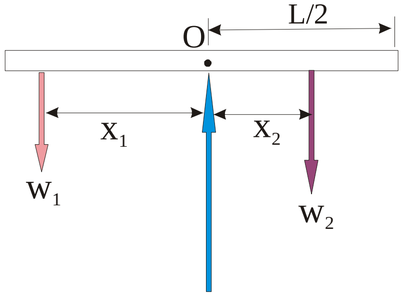
After a little algebra, we get

$$v_H = \sqrt{\frac{2(mg - \tau_B/R)H}{(m + I_O/R^2)}}$$

c. Write the relationships between H and θ_H , and v_H and ω_H . [2pts]

$H = R\,\theta_H$ and $v_H = R\,\omega_H$

7. **[12 pts Total]** You have made a balance scale by using a uniform ruler of length L. You hang it from a hole in its center, the point O. The system is found to balance when the unknown weight W_1 is at x_1 and the known weight W_2 is at x_2 . Find the values of the unknown weight W_1 in terms of W_2, x_1 and x_2 .



Principles [4pts] Details [4 pts] Solve W_1. [2 pts]

$+cw\ \Sigma \tau_O = 0$, $W_2 x_2 - W_1 x_1 = 0$, $W_1 = W_2 (x_2/x_1)$

If W_1 doubles, find the new value of x_1 . [2pts]

One way, there are others. $W_1 \to 2W_1 = 2(W_2 (x_2/x_1) = W_2 (x_2/(x_1/2))$ because $2 = 1/(1/2)$.
Hence $x_1 \to x_1/2$.

8. **[12 pts Total]** The absolute potential energy due to gravity of a small body of mass m a distance R from the center of a large body of mass M, like a planet, is $E_P = -GMm/R$, where G is the Universal Gravitational Constant, $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$. Show that the escape velocity, v_e , of m from M is $v_e = \text{sqrt}(2GM/R)$.

a. What is the total energy of m. [2 pts]

$$E = E_{total} = E_K + E_P = ½ mv^2 - GMm/R\ .$$

b. Obtain the escape velocity. Explain what you are doing. [10 pts]

For m to just escape the gravity of M , the velocity, v, of m must $v \to 0$ as $R = \infty$. Or, equivalently, m it must reach $R = \infty$ with no remaining E_K. So, $E = E_{total} = 0$. That is,

$$½ mv^2 - GMm/R = ½ mv_e^2 - GMm/R = 0\ .$$

So, after a little algebra, one gets $v_e = \text{sqrt}(2GM/R)$.

Final Exam - My Solutions

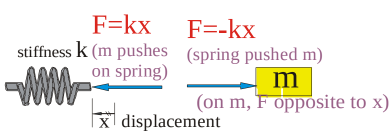
9. **[12 pts Total]S** Total] Simple Harmonic Motion is described mathematically by

$$a\,(acceleration) = -(2\pi f)^2\, x\,(displacement),\ \ in\, calculus,\ \frac{d^2x}{dt^2} = -(2\pi f)^2 x.$$

by definition , a = d^2x/dt^2. Show that the spring of spring constant, or stiffness, k with a mass m attached to it on a horizontal frictionless plane oscillates with a frequency f, where

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

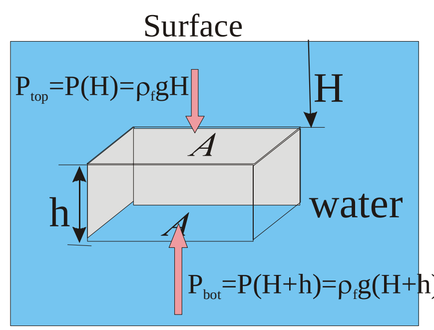
Complete the Free Body Diagram of m and the spring [3pts]	State the Principle(s) [3pts]	Fill in the Details [3pts]	Do It [3pts]



+ -> Σ F = ma ;   -kx = ma .   So, a = -(k/m)x. Comparing with math model gives $(2\pi f)^2$ = k/m. Hence f = (1/2π)sqrt(k/m).

Hooke's Law
F = kx

10. **[12 pts Total]** A rectangular solid body has a height h and top and bottom surfaces of area A, is submerged in water with the top is a depth H under the surface of water, as shown. Explain why the body has a smaller effective weight in water than in air. $\rho_{water} = \rho_f = 1.0\times10^3$ kg/m^3.



a. Sketch on the figure the fluid property that acts on the top and the bottom surfaces.[2 pts]

b. Include their expressions (equations) , on the top and on the bottom. [4pts]

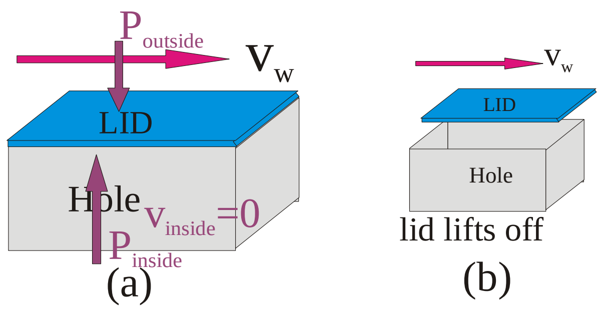
$P_{top} = P(H) = \rho_f\, g\, H.$
$P_{bot} = P(H+h) = \rho_f\, g\, (H+h).$

c. Show that this force F = ρ_f g Ah. Ah = $Volume_{body}$. [6 pts]

$P_{net} = P_{bot} - P_{top} = \rho_f\, g\,(H+h) - \rho_f\, g\, H = \rho_f\, g\, h\,.$

$F = F_{buoyant} = P_{net}\, A = \rho_f\, g\, h\, A$

11. **[12 pts total]** .A hole in the ground is covered by a lid of mass m = 10 kg and surface area A = $1m^2$. A wind of velocity v_w blows over the top of the lid as shown. Find the value of v_w at which the lid lifts off the hole. The density of air is ρ_{air} = 1.2 kg/m^3. g = 9.8 m/s^2



a. On diagram (a) sketch the appropriate pressures. [2 pts]

b. Write the equation relating the pressures . [2 pts]

Bernoulli's Principle

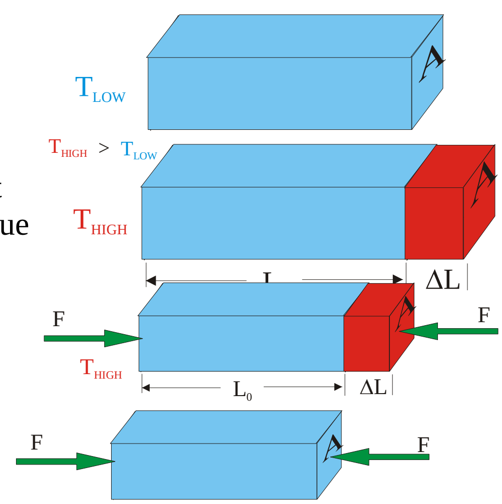
$P_{inside}/\rho_{air} + ½\, v_{inside}^{\ 2} = \ P_{outside}/\rho_{air} + ½\, v_w^{\ 2}$. (1)

c. Show that for lifting the lid is $v_w \geq \sqrt{2mg/(\rho_{air} A)}$. [6pts]

First, the force F on the lid is F = P_{net} A = (P_{inside} - $P_{outside}$) .

Eq (1) , above, using v_{inside} =0, leads to F = $½\rho_{air}\, v_w^{\ 2}$ A .
Next, the force must equal the weight, mg, of the lid,
F >= mg becomes, $½\rho_{air}\, v_w^{\ 2}$ A > = mg.
Hence , after a little algebra, one gets the desired result.

12. **[12 pts total]** . A steel rod a cross sectional area A = $1x10^{-5}m^2$ and an initial length L_0 = 1 m. For steel, the coefficient of linear thermal expansion $\alpha = 1.2x10^{-5}/^oC$ and a Young Modulus $M_Y = 2.0x10^{11}$ N/m^2 .



a. Find the change in length ΔL due to a rise in temperature ΔT = 20^oC. SYMBOLS ONLY (NO NUMBERS). [4 pts]

$\Delta L = \alpha\, L_0\, \Delta T$, by definition of α . (1)

b. At the new temperature, T_{High} , a force F is applied to the rod that pushes the rod back to its original length. Find the stress in the rod due to this force. SYMBOLS ONLY (NO NUMBERS). [8 pts total]
Include Definition of stress. [2 pts]

The definition of stress is , Stress σ = F/A

Include Relation between stress and ΔL. [2 pts]

Stress $\sigma = M_Y \Delta L/L_0'$ by definition of Young's Modulus, M_Y. (2)

Continue [4 pts]

The ΔL from the stress must equal the ΔL from the thermal expansion. Solving (2) for ΔL gives

$\Delta L = \sigma L_0/ M_Y$ (from (2)) = $\alpha\, L_0\, \Delta T$ (from (1))

Let's take both L_0's to be the same yields
$\sigma = M_Y\, \alpha\Delta T$