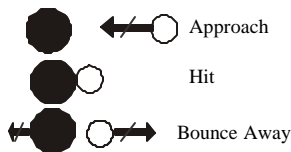


INTRODUCTION

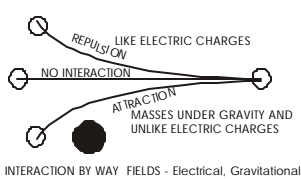
1. What is physics?

One possible description is that physics is a quantitative and causal study of the properties and behaviors of matter and energy, and all interactions between them:

matter<->matter,



DIRECT COLLISIONS



ACTION AT A DISTANCE

Newton's Law of Gravitation

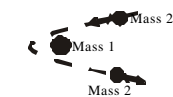
$$F_{\text{Gravity}} \sim \frac{\text{Mass 1} \times \text{Mass 2}}{R^2}$$

Coulomb's Law of Electrostatics

$$F_{\text{Electrostatic}} \sim \frac{\text{Charge 1} \times \text{Charge 2}}{R^2}$$

Masses Attract

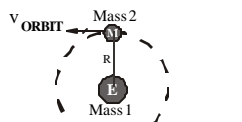
Through Gravity - Newton's Law of Gravitation



Swing Around



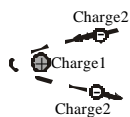
Deflection



The Moon orbiting around the Earth
Circular Orbit

Unlike Charges Attract

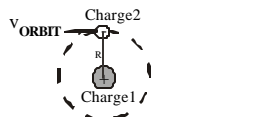
Though Electrostatics - Coulomb's Law



Swing Around



Deflection

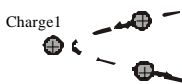


electron orbiting around the proton in the H atom

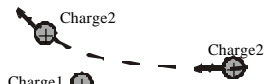
Circular Orbit

Like Charges Repel

Though Electrostatics - Coulomb's Law



Back Scatter



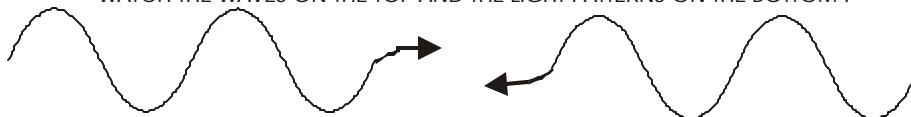
Deflection

No Circular Orbit

DETAILED ACTION AT A DISTANCE

energy<->energy,

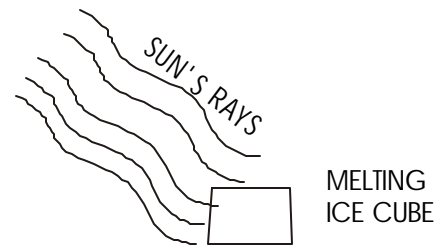
DIP YOUR FINGER IN A SINK OR TUB FILLED WITH WATER,
WATCH THE WAVES ON THE TOP AND THE LIGHT PATTERNS ON THE BOTTOM !



WAVES INTERFERING WITH EACH OTHER

and, finally

matter \leftrightarrow energy

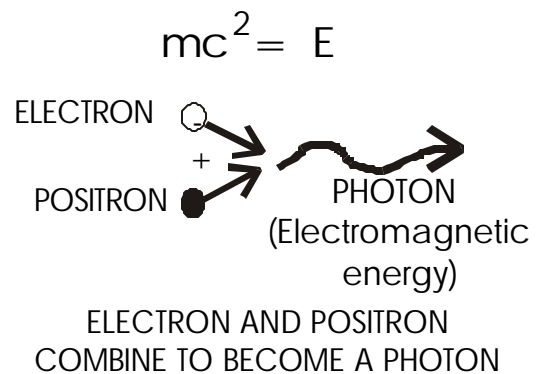
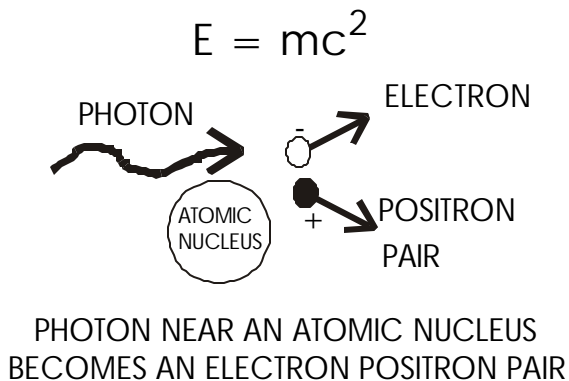


$E = mc^2$, Energy = mass x speed of light squared. Maybe mass and energy are simply different manifestations of the same thing.

c = speed of light = 3×10^8 meters/second,
in a billionth (10^{-9}) of a second (a nanosecond = 1×10^{-9} sec) it travels 0.30m , a little less than 1 foot.

For 1 kg, $E = 1 \text{ kg} \times (3 \times 10^8 \text{ m/s})^2 = 9 \times 10^{16} \text{ kg m}^2/\text{s}^2 = 9 \times 10^{16} \text{ Joules}$
 = 90 petajoules = 90 quadrillion Joules
 = 8.53×10^{13} BTU = 85.3 GigaBTU
 = 2.5×10^{13} Kilowatthours (KwHr) = 25 Tera KwHr.

That's about 1,000 kg for US total annual energy consumption, 4,000 kg for the world.



2. The Physical Description of Physical Entities

All quantitative entities, including physical quantities, have two categories of description, one quantitative, the other qualitative. The quantitative answers “how much” and the qualitative answers the question “of what”. We will start with a very brief discussion of the quantitative, followed by a detailed description of the qualitative.

2.1. **Physical Quantities - Definitions**

Physical objects have several possible quantitative categories depending upon their nature. They are as follows.

Scalars - quantities that are specified by a single magnitude only.

Examples: time, length, speed, volume, mass, density, temperature, heat.

Vectors - quantities that are specified by a single magnitude and a single direction.

The trip is the prototype of a vector.

Examples: displacement, velocity, acceleration, area, force, weight .

Tensors -quantities that are specified by many magnitudes and many directions.

o Scalars are tensors of rank zero.

o Vectors are tensors of rank one.

o Quantities specified by a magnitude and two directions are called tensors of rank two.

Examples of rank two are moment of inertia, stress, and strain . Here we will not deal with the rank two tensor characteristic of these or any other entities. This mention is to simply sound the alert that much more lies beyond.

We explore the characteristics and behavior of vectors in detail later. But, now we want to explore the physical qualitative categories of objects - physical dimensions.

2.2 Physical Dimensions - physical descriptive entities

In describing objects, their conditions and behaviors, we need to determine certain qualitative features of the quantity. These qualitative features are called physical dimensions. First there are five **fundamental physical dimensions**. These are physical qualitative categories that we cannot define, we can only describe their features or characteristics. All other dimensions are called **derived physical dimensions** because they are all derived from the fundamental physical dimensions. The quantitative measures of a dimension are called units.

Some Mathematical Consequences

1. *In symbols, every term in an equation must have the same dimensions.*

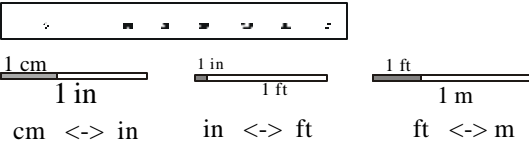
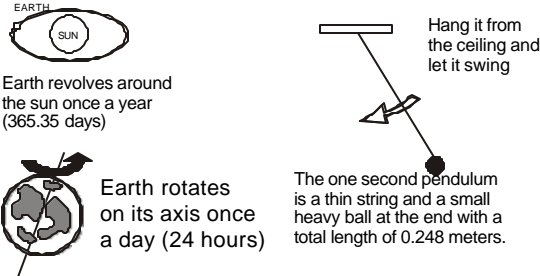
This feature can be used to determine whether an equation is conceptually feasible. If all terms do not have the same dimensions, then the equation cannot describe something that can actually happen. Such an equation is conceptual wrong - **impossible!** If all the terms of an equation do have the same dimensions, it **has a possibility** of describing a real situation, it still may not describe an actual situation - but it has the possibility. As we will see later, this leads to a technique called Dimensional Analysis that can be used as a check on the validity of equations and formula, and to generate new possibilities.



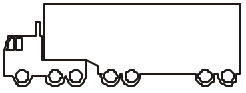

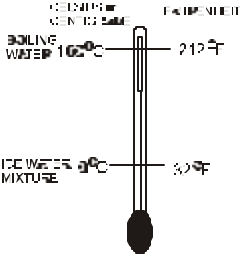
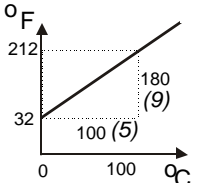
2. *In numerical form, every term in an equation must have the same units.*

This feature makes converting units, also called unit conversion, a very important process. We will deal with that later.

Now let's get to physical dimensions.

2.2.1 Fundamental dimensions -those physical entities that can be described only by example.

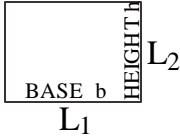
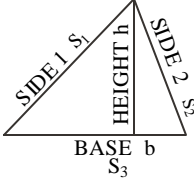
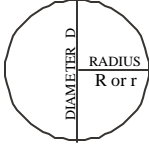
Name	Sym- bol	Dimen- sional Symbol	Description	Units
Length	L	[L]	<p>A measure of size. One stick is "longer" than another.</p>  <p>RADIUS of a nucleus -3 fermi = 3×10^{-15} m, an atom - 3×10^{-10} m Earth 6.38×10^6 m, of Sun 6.96×10^8 m, Earth orbit around Sun 1.50×10^{11} m Milky Way Galaxy -100,000 LY - 10^{21} m HEIGHT of adult human - 1.5 to 2.2 m</p>	<p><u>meter</u>, foot, kilometer, mile, millimeter</p> <p>2.54 cm = 1 in 12 in = 1 f 1mile = 5,280 f 1 km = 1000 m</p> <p>1 light year(LY) = 9.46×10^{15} m</p> <p>1 fermi = 1×10^{-15} m</p>
Time	t	[T]	<p>a measure of progressing change. Sunrise, sunset, sunrise; born, grow up, age and die.</p> 	<p><u>seconds</u>, minutes, hours, days, months, years.</p> <p>60 seconds = 1minute 60 minutes = 1 hour 24 hours = 1day 365.25 days = 1 year 1 century = 100 y 1 millennium =1000y</p>

Mass	m	[M]	<p>The measure of inertia (the ability to resist change in motion).</p> <p> To stop a ping pong ball, no problem;</p> <p> a fast baseball hurts a bit;</p> <p> a big truck, that's a problem, I get all broken up;</p> <p> a huge rolling boulder, I get smashed.</p> <p>Mass of electron 9.1×10^{-31} kg; proton 1.67×10^{-27} kg; dust - 10^{-10} kg; adult human - 50 to 100 kg; a 20 ton truck - 18,130 kg, Earth 6×10^{24} kg; Sun 2×10^{30} kg</p>	<p><u>kilogram (kg)</u>, poundal (rarely used English unit)</p> <p>1 kg weighs 2.206 p on earth.</p> <p>$g=9.81\text{m/s}^2$</p> <p>a 70 kg - 154 p on earth.</p> <p><i>SEE Force, Weight and Gravity below</i></p>
Temp-erature	T	[°]	<p>it's hot; it's cold</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;"> $^{\circ}\text{F} = 9/5^{\circ}\text{C} + 32$ $^{\circ}\text{C} = 5/9 (^{\circ}\text{F} - 32)$ </div> </div> <div style="text-align: center; margin-top: 10px;">  </div> <p>N_2 (nitrogen) liquifies at -210°C (63°K); gold melts 1063°C (1336°K); Sun's surface - 4500°K, interior - 15×10^6 $^{\circ}\text{K}$ to higher (based on model estimates)</p>	<p><u>°K (Kelvin)</u> , $^{\circ}\text{C}$ (Celsius or Centigrade), $^{\circ}\text{F}$ (Fahrenheit), <u>°R (Rankine)</u> $0^{\circ}\text{C} = 273.15^{\circ}\text{K}$ $0^{\circ}\text{K} = -273.15^{\circ}\text{C}$ $0^{\circ}\text{F} = 459^{\circ}\text{R}$</p>
Charge	q	[Q]	it's electrifying.	<u>C (Coulomb)</u> ,

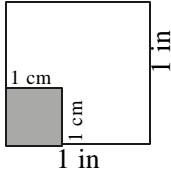
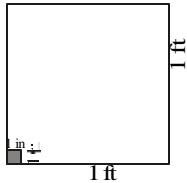
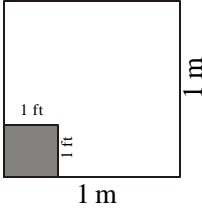
2.2.2 Derived dimensions are those derived from, hence expressed in terms of, the fundamental dimensions.

Area - length x length [L²]

Units: sq. meter (m²), sq.ft (ft²)

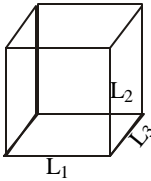
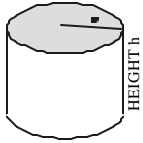

			
AREA A=	$L_1 \times L_2$ HEIGHT x BASE	$\frac{1}{2}$ HEIGHT x BASE	$\pi R^2 = (D/4)^2 \pi$
PERIMETER P=	$2(L_1 + L_2)$ 2(HEIGHT+BASE)	$S_1 + S_2 + S_3$	$2\pi R = D\pi$

Comparisons of some areas

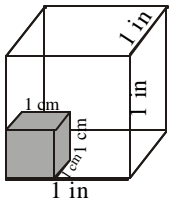
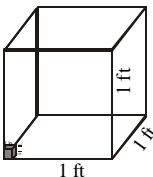
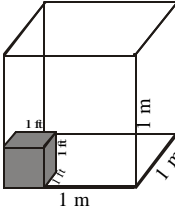
		
$cm^2 \leftrightarrow in^2$	$in^2 \leftrightarrow ft^2$	$ft^2 \leftrightarrow m^2$

Volume - length x length x length [L³]

Units: cubic meter (m³), cu.ft (ft³)

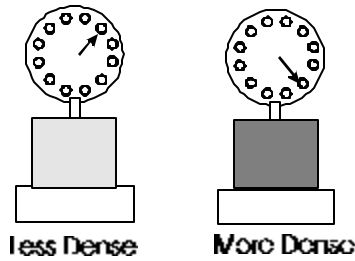
			
VOLUME V=	$L_1 \times L_2 \times L_3$	$\pi R^2 h$	$\frac{4}{3} \pi R^3$
SURFACE AREA A=	$2(L_1 L_2 + L_1 L_3 + L_2 L_3)$	$2(\pi R^2 + Rh)$	$4\pi R^2$

Comparisons of some volumes

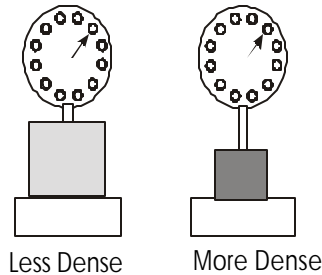
		
$cm^3 \leftrightarrow in^3$	$in^3 \leftrightarrow ft^3$	$ft^3 \leftrightarrow m^3$

Density

**SAME SIZE
DIFFERENT
SCALE READINGS**



**SAME
SCALE READINGS
DIFFERENT SIZES**



MATERIAL	mass density kg/m ³	weight density p/f ³
air (1atm,20 C)	1.20E+00	7.52E-02
ethanol	7.10E+02	5.08E+01
gasoline	6.78E+02	4.25E+01
wood pine	4.79E+02	3.00E+01
ice	9.20E+02	5.76E+01
water (4 C)	1.00E+03	6.27E+01
seawater (4 C)	1.03E+03	6.45E+01
blood	1.06E+03	6.64E+01
aluminum	2.70E+03	1.69E+02
iron	7.20 E+03	4.50E+02
lead	1.13E+04	7.08E+02

Density is a specific mass and/or weight. One is mass per unit volume, specific mass;

$$\text{mass density } \rho_{\text{mass}} = \text{mass/volume} = m/\text{vol.}$$

The other is weight per unit volume, specific weight;

$$\text{weight density } \rho_{\text{weight}} = \text{weight/volume} = w/\text{vol} = \text{mg/vol.}$$

From here on we will use the symbol ρ for both types of density. Which one applies in a given situation is usually obvious from the context of the situation.

The pictures visually display the meaning. On the top, both items having the same volume. But one is heavier than the other as indicated by the scale readings. The heavier one is more dense - has the higher density ($m/\text{vol} > m/\text{vol}$). On the bottom, the two items have the same mass (weight) as indicated by the scale readings. But, they have different volumes as illustrated by the sizes. The smaller one (smaller volume) is more dense - has the higher density ($m/\text{vol} > m/\text{VOL}$).

Every material has a characteristic value of density at any given temperature and pressure. On the other hand, several materials can have the same density under the same conditions. For example, you could make an alloy of lead and aluminum that has the same density as iron. In mathematical terms, density is not a one-to-one mapping, density \leftrightarrow material. It is a one-to-many mapping, a density can correspond to many materials, but a given material corresponds to one unique value of density.

All materials change volume with changing temperature and pressure. Hence density values are specified for a given temperature and pressure. Unless otherwise stated, the values are for standard temperature and pressure - 15 °C and 1.0 atmosphere.

The above table lists the densities of a few materials. It is intended to show the range of densities rather than being exhaustive.

2.3 Units and Conversion of units

2.3.1 Introduction

Every term in an equation must not only have the same physical dimensions when in symbolic form, but also must have the same units when in numerical form. So converting from one unit to another is a very important task. And one that you will frequently encounter. There are a large number of conversions that are strictly proportional to each other. That is conversions that do not have an additive constant like the conversion between Fahrenheit and Centigrade degrees ($^{\circ}\text{F} = 9/5 \text{ }^{\circ}\text{C} + 32$). All of these conversions are always done by a sequence of successive proportions. Methods differ in the details of how they handle the proportions. The basic methods are : 1) a sequence of independent successive proportions, 2) a sequence of linked successive proportions, and 3) a sequence of inline successive proportions. Here we develop the method of a sequence of inline successive proportions and compare it with the other two methods. We find that checking the correctness of the conversion is very easy using this method as compared to the other two methods.

This method has the following advantages;

1. It is programmatic. That is before using any numbers you lay out a program to get from the units you have to the units you want. Only after completing the program do you use the numbers.
2. It is “self correcting” in the sense that errors are made obvious and easy to correct. The steps are as follows.

Any conversion in the form of

$$a_1 * \text{unit1} = a_2 * \text{unit2} ,$$

where a_1 and a_2 are numerical constants, is a proportion of the form

$$\text{unit1}/\text{unit2} = a_2 / a_1 \text{ and the reciprocal } \text{unit}_2 / \text{unit}_1 = a_2/a_1 .$$

For example,

$$1\text{f} = 12 \text{ in}$$

is the proportion

$$\text{in}/\text{f} = 1/12 \text{ “ an inch is to a foot as 1 is to 12”},$$

and the reciprocal

$$\text{f}/\text{in} = 12/1 \text{ “ a foot is to an inch as 12 is to 1” } .$$

But, a more interesting form of these proportions are the pairs

$$a_1 \text{unit1}/a_2 \text{unit2} = 1 \quad \text{and} \quad a_2 \text{unit2}/a_1 \text{unit1} = 1,$$

$$1 \text{ f}/12 \text{ in} = 1 \quad \text{and} \quad 12 \text{ in}/1 \text{ f} = 1 .$$

This forms the base of the method of unit conversion developed here. To see why, let’s look at the following example.

$$1 f = 12 \text{ in} \quad \text{yields} \quad 1 f / 12 \text{ in} = 1 \quad \text{and} \quad 12 \text{ in} / 1 f = 1;$$

$$1 \text{ mi} = 5280 \text{ f} \quad \text{yields} \quad 1 \text{ mi} / 5280 \text{ f} = 1 \quad \text{and} \quad 5280 \text{ f} / 1 \text{ mi} = 1.$$

The equation

$$1 = 1 * 1$$

is a trivial truism (from the definition of the unit element under multiplication). But, looking at the conversion proportions above, we see there are four possible ways of multiplying the (f:in) with the (f:mi) conversion. They are

$$1 = \frac{12 \text{ in}}{1 f} * \frac{5280 \text{ f}}{1 \text{ mi}} = \frac{12 \text{ in}}{1 f} * \frac{1 \text{ mi}}{5280 \text{ f}} = \frac{1 f}{12 \text{ in}} * \frac{1 \text{ mi}}{5280 \text{ f}} = \frac{1 f}{12 \text{ in}} * \frac{5280 \text{ f}}{1 \text{ mi}}.$$

The first and third are very interesting.

$$1 = \frac{12 \text{ in}}{1 f} * \frac{5280 \text{ f}}{1 \text{ mi}} = \frac{12 * 5280 \text{ in}}{1 * 1 \text{ mi}} \quad \text{and} \quad 1 = \frac{1 f}{12 \text{ in}} * \frac{1 \text{ mi}}{5280 \text{ f}} = \frac{1 * 1 \text{ mi}}{12 * 5280 \text{ in}} \quad (1)$$

The unit f (foot) has cancelled in each yielding

$$1 = 12 * 5280 \text{ in} / \text{mi} = 69840 \text{ in} / \text{mi} \quad \text{and} \quad 1 = 1 / 69840 \text{ mi} / \text{in},$$

respectively. If we now cross multiply each, we get

$$1 \text{ mi} = 69840 \text{ in} \quad \text{and} \quad 1 \text{ in} = (1 / 69840) \text{ mi},$$

respectively. This suggests that we can make unit conversions by judiciously multiplying 1 by itself as many times as necessary to eliminate the initial unit and leave only the desired new unit.

Let's follow this lead to solve the following:

We want to find how many kilometers (km) are in 5 miles (mi). That is $5 \text{ mi} = ? \text{ km}$. Let's use the following: $1 f = 12 \text{ in}$, $1 \text{ m} = 39.37 \text{ in}$ and $1 \text{ km} = 1000 \text{ m}$.

First let's write all of the "=1" proportion that result from the above unit conversions. We get

$$1 = \frac{1 \text{ mi}}{5280 \text{ f}} = \frac{5280 \text{ f}}{1 \text{ mi}}, \quad 1 = \frac{1 f}{12 \text{ in}} = \frac{12 \text{ in}}{1 f}, \quad 1 = \frac{1 \text{ in}}{39.37 \text{ m}} = \frac{39.37 \text{ m}}{1 \text{ in}}, \quad 1 = \frac{1000 \text{ m}}{1 \text{ km}} = \frac{1 \text{ km}}{1000 \text{ m}}.$$

Following the suggestion of Equation (1), let's judiciously multiplying 1 by itself four time so that km is on top. Start by putting the proportion that has km on top. This is the proportion that has *km* on top and *m* on bottom. But, that gives us an *m* to get rid of.

$$1 = \frac{1 \text{ km}}{1000 \text{ m}}$$

Now we have an *m* to cancel. So, we multiply this on the left by a "1" that will that

cancel m . But, that gives an in to get rid of.

$$1 = 1 * \frac{1 \text{ km}}{1000 \text{ m}} = \frac{1 \text{ m}}{39.37 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}}$$

Now we have an in to cancel. So, we multiply this on the left by a "1" that will that will cancel in . But, that gives us a f to get rid of.

$$1 = 1 * \frac{1 \text{ m}}{39.37 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}} = \frac{12 \text{ in}}{1 \text{ f}} * \frac{39.37 \text{ m}}{1 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}}$$

Now we have an f to cancel. So, we multiply this on the left by a "1" that will that will cancel f . At last we a mi on the bottom which will be cancelled by our starter or initial unit, mi .

$$1 = 1 * \frac{12 \text{ in}}{1 \text{ f}} * \frac{1 \text{ m}}{39.37 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}} = \frac{5280 \text{ f}}{1 \text{ mi}} * \frac{12 \text{ in}}{1 \text{ f}} * \frac{1 \text{ m}}{39.37 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}} \quad (2)$$

Finally, multiplying both sides by 5 mi gives the desired result

$$5 \text{ mi} = 5 \text{ mi} \left[\frac{5280 \text{ f}}{1 \text{ mi}} * \frac{12 \text{ in}}{1 \text{ f}} * \frac{1 \text{ m}}{39.37 \text{ in}} * \frac{1 \text{ km}}{1000 \text{ m}} \right] \quad (3)$$

The end result of this is

$$5 \text{ mi} = 5 \text{ mi} * \frac{5280 * 12 \text{ km}}{39.37 * 1000 \text{ mi}} = 5 * 1.609 \text{ km} = 8.045 \text{ km} \quad .$$

Notice that the stuff inside the square brackets of Eq. (3) equals 1. Separating the units and the numbers gives

$$5 \text{ mi} = 5 \text{ mi} \left[\left(\frac{5280}{1} \right) \frac{\text{f}}{\text{mi}} \left(\frac{12}{1} \right) \frac{\text{in}}{\text{f}} \left(\frac{1}{39.37} \right) \frac{\text{m}}{\text{in}} \left(\frac{1}{1000} \right) \frac{\text{km}}{\text{m}} \right] \quad (4)$$

Now let's take out the conversion numbers gives

$$5 \text{ mi} = 5 \text{ mi} \left[\left(\quad \right) \frac{\text{f}}{\text{mi}} \left(\quad \right) \frac{\text{in}}{\text{f}} \left(\quad \right) \frac{\text{m}}{\text{in}} \left(\quad \right) \frac{\text{km}}{\text{m}} \right] \quad . \quad (5)$$

What we have here is the sequence of inline consecutive ratios that successively cancel units, starting with the initial unit and leaving only the desired final unit as the end result. The ratios derive from their respective unit conversion proportions.

From this we see that we can do unit conversions in three basic steps

1. Step One consists of knowing where you are and where you want to get to, and deciding

on how to make the trip. This step builds the program by way of a sequence of inline consecutive ratios that successively cancel all unit, starting with the initial unit leaving as the end result the desired final unit. But, **NO NUMBERS YET!!**

2. Step Two consists of inserting in the appropriate conversion numbers in their correct positions.
3. Step Three is checking the position of conversion numbers.

We will see by the examples below how easy it is to check the correctness of your conversion as compared to when done in the frequently used methods of successive independent proportions or successive linked proportions.

2.3.2 An Example

Let's do the above conversion as an example to illustrate the method.

How many km are there in 5 mi.? That is, 5 mi = ? km . Use the following conversions: 1mi = 5280 f, 1 f = 12 inch, 1 inch = 2.54 cm, 1m = 100 cm and 1 km = 1000m..

Step One: Layout the program.

First, you might want to set up a high level **pseudo-code**, shown below, to outline the “trip” from your initial unit, mi, to the final unit, km using the given unit conversions .

mi->f, f->inch, inch-> cm, cm -> m, and finally m -> km,
or shorter mi -> f -> in -> cm -> m -> km.

This is a sequence that lays out the steps we want to execute without the details of how to do it. All conversion methods would use this same pseudo-code for this problem. But, the details of its implementation vary.

The program follows the methodology that lead to Eq. (5). We set up the sequence of inline consecutive ratios, the ratios result from the respective conversion proportions, that successively cancel units, starting with the initial unit and leaves only the desired final unit as the end result to accomplish the objectives laid out in the pseudo-code above.

$$mi \left(\right) \frac{f}{mi} \left(\right) \frac{in}{f} \left(\right) \frac{m}{in} \left(\right) \frac{km}{m} \tag{6}$$

Correctness :

This step is automatically correct because it forces you to do what has to be done!!!

Step Two: **Now** enter the numbers for each of the conversions.

$$mi(5280) \frac{f}{mi} (12) \frac{in}{f} (2.54) \frac{cm}{in} \left(\frac{1}{100} \right) \frac{m}{cm} \left(\frac{1}{1000} \right) \frac{km}{m} \tag{7}$$

Step Three: **check** for the correctness of the numerical entries.

For example, substituting (5280f=mi) , the first term cancels to 1.

$$mi(5280) \frac{f}{5280 = mi} (12) \frac{in}{12in = f} (2.54) \frac{cm}{2.54cm = in} \left(\frac{1}{100}\right) \frac{m = 100}{cm} \left(\frac{1}{1000}\right) \frac{km = 1000}{m} \quad (8)$$

Suppose we mistakenly had written

$$(100) \frac{m}{cm}, \text{ doing the checking gives } (100) \frac{m(=100)}{cm} \quad (9)$$

This does not cancel. The checking tells us that the number is upside down. So correct it.

Finishing it gives

$$5mi = 5 \times \left(\frac{5280 \times 12 \times 2.54}{100 \times 1000} \right) km = 5 \times 1.609 km = 8.047 km \quad (10)$$

2.3.4 Another Example - Choices

Let's get **5 mi = ? km** using the conversions

$$1f = 12 in, 1m = 39.37in \text{ and } 1km = 1000 m.$$

We did this in establishing the method. Comparing Eqs. (5) and (6) we see that specific program strictly depends on the intermediate unit conversions used.

2.3.6 The Other Method - The Method of Series of Individual Proportions

For any given conversion sequence, all methods follow the same pseudo-code. But, different methods handle the details differently. For example, below we have what I will call a sequence of linked successive proportions.

$$5mi = 5 \times 5280 f = 5 \times 5280 \times 12 in = \frac{5 \times 5280 \times 12}{39.37} m = \frac{5 \times 5280 \times 12}{39.37 \times 1000} km = 8.047 km$$

There are several questions here. The first is how do you check the correctness of the answer - both in the logic of the steps and placement of the numbers? Here there is no record of what each step does, only the result. Hence there is no way to check either the logic or the number placements without actually redoing it. Catching an error is by luck. Let's examine another approach.

$$5mi = 5 \times 5280 \frac{f}{mi} = 5 \times 5280 \times 12 \frac{in}{f} = \frac{5 \times 5280 \times 12}{39.37 \frac{in}{m}} = \frac{5 \times 5280 \times 12}{39.37 \times 1000 \frac{m}{km}} = 8.047 km$$

This last layout is an improvement because each step shows what conversion is made. It is approaching the program method. There is still no way of checking without simply redoing it. And the denominator

conversions can be a bit confusing.

The worst case is where each proportion conversion is done as a sequence of separate independent steps. For instances;

$$5mi = 5 \times 5280 \frac{f}{mi} = 26400f$$

then

$$26400f = 26400 \times 12 \frac{in}{f} = 316800in$$

next

$$316800in = 316800 / 39.37 \frac{in}{m} = 8046.7m$$

and so on. This is more tedious and again can only be checked by redoing it.

2.3.6 Some more examples

1. mph to in/s

Step One: Layout the Program

Find 60 mph = ? f/s using 1 mi = 5280 f, 1 f = 12 in, 1 hr = 60 min, 1 min = 60 s.

Pseudo-code 1) mi -> f, f -> in; 2) hr -> min, min -> s.

$$60mph = 60 \frac{mi \left(\frac{f}{mi} \right) \left(\frac{in}{f} \right)}{hr \left(\frac{min}{hr} \right) \left(\frac{s}{min} \right)}$$

Step Two: Fill in the numbers

$$60mph = 60 \frac{mi(5280) \frac{f}{mi} (12) \frac{in}{f}}{hr(60) \frac{min}{hr} (60) \frac{s}{min}}$$

You finish it.

2. Newton (N) -> Pound (p) Conversion

To convert between Newton (the International [metric] standard for force) and the pound (the English - U.S. unit for force) requires Newton's 2nd Law of Motion, F = ma. The reason for this is as

follows. The International System of Measures uses the fundamental dimension mass in the unit of kilogram as its standard. In this system force is a derived dimension and its unit is the Newton. The English system uses force as its fundamental dimension by way of weight in the unit of pound (p). The relation between the two units, kg and p, along with g, must be directly measured. On earth $g = 9.81 \text{ m/s}^2$ and 1kg weighs 2.206 p. This result of this measurement depends on location. Since m is constant everywhere, then (W/g) is constant. For instance, on the moon both g and W are about 1/6th there values on earth so the result is true everywhere. we now travel the path laid out in this annotated pseudo-code

$$N \text{ (force)} \quad \rightarrow \quad kg \text{ (} m = F/g = W/g \text{)} \quad \rightarrow \quad p \text{ (force)} \quad .$$

The program with conversion numbers is

$$1N = 1N \left(\frac{kg}{N} \right) \left(\frac{p}{kg} \right) = 1N \left(\frac{1}{9.81} \right) \frac{kg}{N} (2.206) \frac{p}{kg} = 0.2249p$$

So $1 N = 0.2249 p$ and its reciprocal gives $1 p = 4.447 N$.

Further we have $1kg \times 9.81 \text{ m/s}^2 = 9.81 N$. Thus 1 kg weighs 9.81 N.

3. Joules -> fp (foot pound) Conversion.

Here we use the initial data $kg = 9.81N$, $1 kg = 2.206p$ on earth, $1m = 100cm$, $1in = 2.54 cm$, and July 26, 20001 $f = 12 in$

$$1J = 1Nm = 1N \times m = 1N \left(\frac{1}{9.81} \right) \frac{kg}{N} (2.206) \frac{p}{kg} \times m (100) \frac{cm}{m} \left(\frac{1}{2.54} \right) \frac{in}{cm} \left(\frac{1}{12} \right) \frac{f}{in} = 0.739 fp$$

2.3.6 Exercises:

1. Show that 3 mph (miles per hour = mi/hr) = 4.40 f/s, = 1.34 m/s, = 4.82 km/h.
Use $1mi = 5,280f$, $1f = 12in$, $1in = 2.54 cm$, $1m = 100cm$, $1km = 1,000m$.
2. Show that $1 f^2$ (1 square foot) = $144 i^2$, = $929.0 cm^2$ = $0.0929 m^2$
Use the units listed above.

3. Scientific Notation and the Size of Things

3.1 Scientific Notation

Physics is a quantitative science. Every observation, every experiment, every theory, in the end must produce numbers. Every theory must produce numbers that actually occur in observations or experiments. New observations demand new theories to explain them. A new theory drives new experiments to test the new theory - how close do the numbers produced by the theory match those of the experiment or observation? There is a constant drive for ever increasingly refined measurement tools and methods - increased accuracy and increased precision. Since numbers are so important we must have the most effective and efficient ways to represent numbers. That's called **scientific notation**.

Scientific notation separates a numerical quantity into two parts - **significant figures** and **order of magnitude**. The number 1,234,872,492,091, or to four figures 1.235 billion is represented as

$$\begin{array}{cc}
 1.235 \times 10^9. \\
 / \qquad \backslash \\
 \textit{significant} & \textit{order of} \\
 \textit{figures} & \textit{magnitude}
 \end{array}$$

The **order of magnitude** is powers of ten. It tells us how big the units. The **significant figures** is a digital part usually with one figure to the left of the decimal point. It tells us how many of those units. Sometimes it is written in the exponential form as

$$1.235 \text{ E}+09 ,$$

where the "E+09" says that the order of magnitude is +9. Notice that our language carries this notation when we say 1.235 billion. In the long way of "1 billion, 2 hundred and 35 million", the 1 billion leads.

Let's look at an example. Suppose you found out that a body from outer space was coming into the earth at 90,000 mph and was to crash where you are sitting or standing right now. Compare your concerns and reactions when you are told that its mass is $9.3 \times 10^{+9}$ kg; its mass is 9.3×10^{-9} kg. Which number catches your attention? At first do you really care about the 9.3 here?

The scientific notation makes computations easier because the significant figures and the orders of magnitude are handled separately. The handling of the significant figures depends on the nature of the numbers. There are exact numbers and measured numbers. An exact number has arbitrary accuracy. On the other hand, the accuracy of measured numbers is limited by the precision of the measuring instruments and the accuracy of the measuring techniques. So, the number of significant figures retained in calculations is different for the two, in fact opposite. For exact numbers the maximum number of significant figures are retained. But, for measured numbers, the smallest number of significant figures are retained. We will illustrate this in the examples below.

For addition and subtraction we deal only with the significant figure after all terms have been converted to same order of magnitude. Then readjust order of magnitude to put the significant figure in standard form.

$$\begin{array}{l}
 1.235 \times 10^9 + 2.831 \times 10^7 = 1.235 \times 10^9 + (2.831/100) \times (10^7 \times 100) \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{divide by} \quad \text{multiply by} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad 100 \text{ to get order of magnitude the same}
 \end{array}$$

for exact numbers

$$\begin{array}{l}
 = 1.235 \times 10^9 + 0.02831 \times 10^9 = 1.26331 \times 10^9 , \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{keep all.}
 \end{array}$$

But, for measured numbers

$$\begin{array}{l}
 = 1.235 \times 10^9 + 0.02831 \times 10^9 = 1.263 \times 10^9 , \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{keep only the smallest number} \\
 \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{of significant figures.}
 \end{array}$$

$$\begin{array}{l}
 1.235 \\
 0.02831 \text{ lost, dropped}
 \end{array}$$

1.263~~31~~ gone!

Or, alternatively,

$$1.235 \times 10^9 + 2.831 \times 10^7 = 123.5 \times 10^7 + 2.831 \times 10^7 = 126.3 \times 10^7 = 1.263 \times 10^9$$

For multiplication and division we operate on the two parts separately, then make order of magnitude adjustments after all is done. Operating with the order of magnitude is particularly simple - it follows the laws of exponents ,

$$a^m * a^n = \overset{\text{m- times}}{(a \dots a)} * \overset{\text{n-times}}{(a \dots a)} = a^{m+n} \quad \text{and} \quad a^m / a^n = \overset{\text{m- times}}{(a \dots a)} / \underset{\text{n-times}}{(a \dots a)} = a^{m-n}$$

For examples,

$$1. \quad 1.235 \times 10^9 * 8.32 \times 10^4 = 10.275 \times (10^{9+4}) = 1.0275 \times 10 \times (10^{13}) = 1.0275 \times 10^{14}$$

for exact numbers.

$$1.03 \times 10^{14}, \quad \text{for measured numbers.}$$

$$2. \quad 1.235 \times 10^9 / 8.32 \times 10^4 = (1.235 / 8.32) \times (10^{9-4}) = 0.148375 \times (10^5) = 1.48375 \times 10^{-1} \times 10^5$$

$$= 1.48375 \times 10^4 \quad \text{for exact numbers.}$$

$$= 1.48 \times 10^4 \quad \text{for measured numbers.}$$

Examples of exact numbers

Number like $\frac{1}{2}$, $\frac{1}{3}$, 100, e, π are examples of exact numbers. Even though the values of e, π and other such numbers are usually listed with a limited number of digits, their values can be calculated to as many digits as you want. Both e and π have been calculated millions of digits.

Examples of measured numbers

All physical constants are measured numbers. In the United States, it is the mission of the National Bureau of Standard to maintain and improve the accuracy of all of the numbers. They are always fighting to get values by one more significant figure. When the 10 significant figure is secured, they go for the eleventh, and so on.

Rounding

Rounding is best explained by example. Given the number

3.4652754	
3.465275	rounded to 7 digits
3.46528	rounded to 6 digits
3.4653	rounded to 5 digits
3.465	rounded to 4 digits
3.47	rounded to 3 digits
3.5	rounded to 2 digits
4.	rounded to 1 digit.

5 or larger go up by 1. Less than 5 drop down by 1.

3.2 Power of 10

The BIG, the LARGE

<u>unit name (prefix)</u>	<u>unit value</u>	<u>power of 10</u>	<u>Exponential</u>
units	1 = 10/10	10^0	1.0E+00
tens (deca-)	10 = 1*10	10^1	1.0E+01
hundreds (hecto-)	100 = 1*10*10	10^2	1.0E+02
thousands(kilo-)	1,000 = 1*10*10*10	10^3	1.0E+03
ten thousands	10,000 = 1*10*10*10*10	10^4	1.0E+04
hundred thousands	100,000 = 1*10*10*10*10*10	10^5	1.0E+05
millions (mega-)	5, 0's <--10, 5 times > 1,000,000 = 1*10*10*10*10*10*10	10^6	1.0E+06
ten millions	6, 0's <--10, 6 times ---> 10,000,000 = 1*10*10*10*10*10*10*10	10^7	1.0E+07
hundred millions	7, 0's <----10, 7 times ----> 100,000,000 = 1*10*10*10*.....*10*10*10	10^8	1.0E+08
billions (giga-)	8, 0's <-----10, 8 times----> 1,000,000,000 = 1*10*10*.....*10*10	10^9	1.0E+09
ten billions	nine 0's <----10, 9 times ----> 10,000,000,000 = 1*10*.....*10	10^{10}	1.0E+10
hundred billions	ten 0's <---10 times --> 100,000,000,000 = 1*10*.....*10	10^{11}	1.0E+11
trillions (tera-)	eleven 0's <---11 times --> 1,000,000,000,000 = 1*10*.....*10	10^{12}	1.0E+12
ten trillions	twelve 0's <---12 times --> 10,000,000,000,000 = 1*10*.....*10	10^{13}	1.0E+13
hundred trillion	thirteen 0's <---13 times --> 100,000,000,000,000=10*10*10*.....10*10*10	10^{14}	1.0E+14
quadrillions (peta-)	fourteen 0's <---10, 14 times --> 1,000,000,000,000,000 = 10*10*.....*10*10 fifteen 0's <---10, 15 times -->	10^{15}	1.0E+15

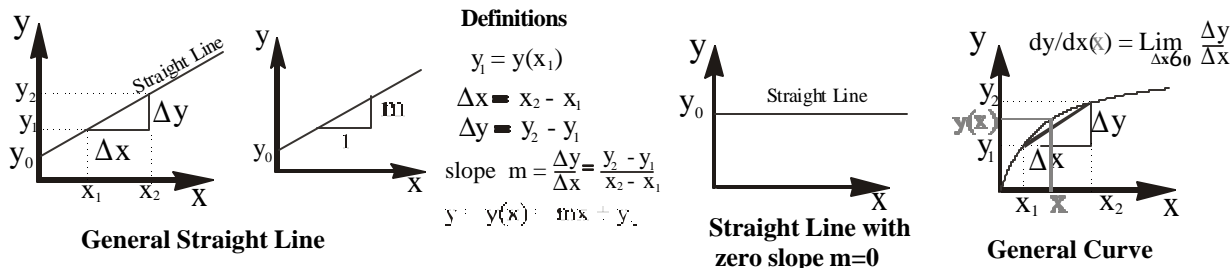
The SMALL, the TINY of IT ALL - the LAND of “THS”

<u>unit name (prefix)</u>	<u>unit value</u>	<u>power of 10</u>	<u>Exponential</u>
tenths (deci-)	0.1=1/10	10^{-1}	1.0E-01
hundredths (centi-)	0.01=1/100	10^{-2}	1.0E-02
thousandths(milli-)	0.001=1/1,000	10^{-3}	1.0E-03
ten thousandths	0.0001=1/10,000	10^{-4}	1.0E-04
hundred thousandths	0.00001= 1/100,000	10^{-5}	1.0E-05
millionths (micro-)	4, 0's 0.000001= 1/1,000,000	10^{-6}	1.0E-06
ten millionths	5, 0's 0.0000001= 1/10,000,000	10^{-7}	1.0E-07
hundred millionths	6, 0's 0.00000001= 1/100,000,000	10^{-8}	1.0E-08
billionths (nano-)	7, 0's 0.000000001=1/1,000,000,000	10^{-9}	1.0E-09
ten billionths	8, 0's 0.0000000001=1/10,000,000,000	10^{-10}	1.0E-10
hundred billionths	nine 0's 0.00000000001=1/100,000,000,000	10^{-11}	1.0E-11
trillionths (pico-)	ten 0's 0.0000000000001=1/1,000,000,000,000	10^{-12}	1.0E-12
ten trillionths	eleven 0's 0.00000000000001= 1/10,000,000,000,000	10^{-13}	1.0E-13
hundred trillionths	twelve 0's 0.000000000000001=1/100,000,000,000,000	10^{-14}	1.0E-14
quadrillionths (femto-)	thirteen 0's 0.0000000000000001=1/1,000,000,000,000,000	10^{-15}	1.0E-15
	fourteen 0's		

4. Some Key Math Concepts and Manipulations

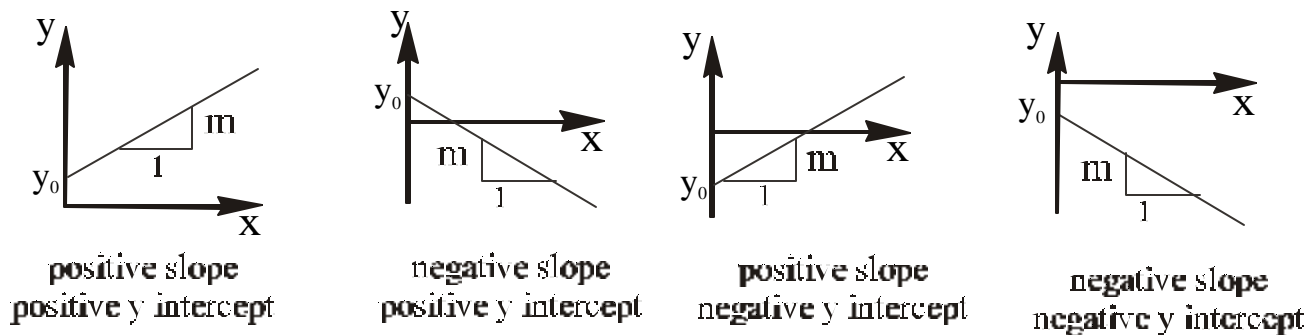
Here we discuss some math concepts and manipulations that are key to this course. The basic calculus (ca 1600 CE) concepts of derivative and integral derive from the simple geometric concepts of slope and area (ca 1500-200 BCE), respectively. All too frequently, neophyte students forget these basic concepts as they exclusively fight with, and concentrate on, the mathematical manipulations of getting the derivative and integral by way of the formulas. In this course we will directly use the concepts of slope and area as base of the derivative and the integral. There are two reasons for this. One is that in many cases the slope and area are obvious from geometry without the calculus (ca 1600 CE). Two, in the case of areas you must have an analytic function to take an integral. In many experimental situations one has only an irregular graph to work with having no analytic function which are required for differentiation or integration. So, other methods must be used.

4.1 Slopes and Derivatives



1. Every curve has a slope.
2. A **straight line** has a **constant slope** and an intercept y_0 on the y axis as illustrated in the figure.
3. If the function $y(x)$ is a continuous differentiable function of x and you know $y(x)$ then the slope at the point x can be found mathematically by taking the derivative at x , $dy/dx(x)$. **See “Derivatives and Differentiation”.**
4. If $y(x)$ is not known, then one must use graphical methods.

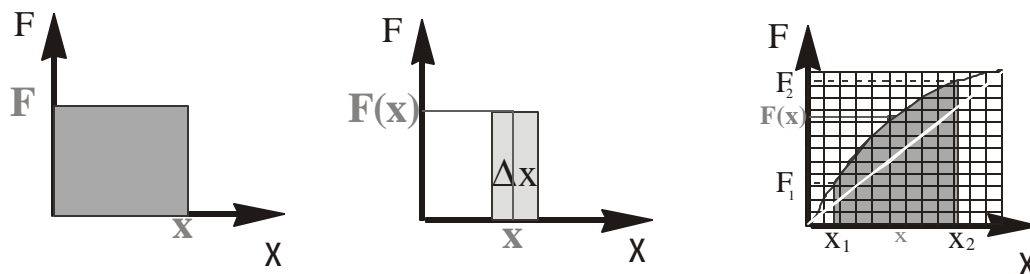
More examples of straight lines



Exercise 4.1: Sketch the straight lines $y = 3x + 4$, $y = 3x - 4$, $y = -3x + 4$, $y = -3x - 4$.

4.2 Products, Areas, Area Elements and Integrals

We will use these ideas constantly - i.e. displacement, velocity, work, energy

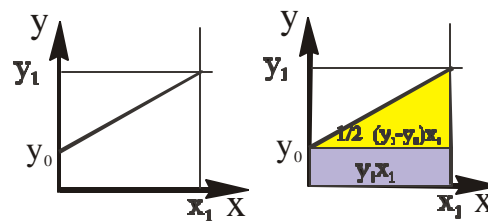


Effect of F over x is the area under the F vs x curve

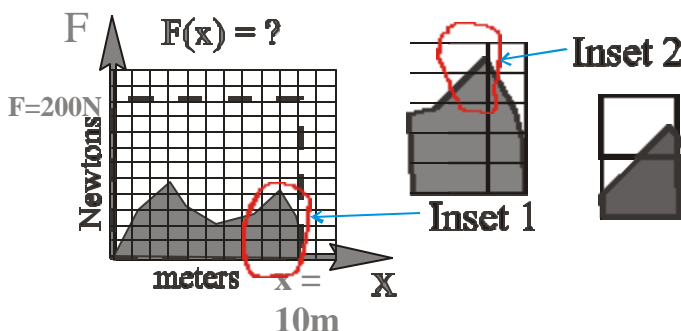
- Every **product** can be viewed as an **area**. For example, consider the product Fx , where F is force and x is for displacement parallel to the force. Set up the F and x axes. Then for constant F and constant x , the **product Fx** is the **rectangular area** F (units = N, here) high and x (units = m, here) wide, as shown in the first figure above. Physically, this area equals the work done by the constant force F over the parallel displacement x . Work has the unit $(Nm) = J$ (Joules).
- This can lead us to a $F(x)$ situation as shown in the middle figure. $F\Delta x$ is an area element of the $F(x)$ graph. Physically, it is an element of work, $\Delta Work = F\Delta x$.
- An area element means that the total effect over the range x_1 to x_2 is the area as shown in the rightmost figure above. This is the basis of the definite integral.

This area can be found by a number of methods.

- If the curve is a straight line, the area is a triangle, rectangle or a combination of the two. The area can be found geometrically using the formulas for the areas of rectangles and triangles.
- If the curve is non-linear(not a straight line) and you have the function $F(x)$, you can use integration to get the area. **See "Areas and Integration"**.
- If you only have a graph and no identifiable mathematical function $F(x)$ then you might use one of these two methods (1) the graphical method of **"counting the squares"**, or (2) by **"cutting and weighing"**.

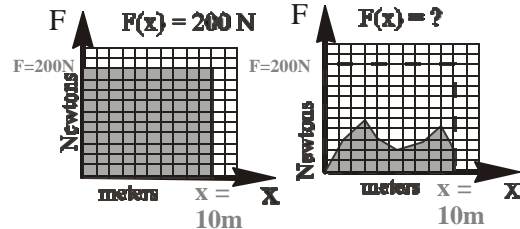


(1) For **counting the squares**, put the curves on a rectangular grid. Then count the number of squares covered by the curve. Set the values on the axes as shown in the example on the right. Note that on the vertical there are 10 grid lines for 200N, hence a height of 20N per grid. On the horizontal axis



there are 10 grid lines for 10 m, hence a width of 1 m per grid. So each grid square is $200\text{Nm}=200\text{J}$. Inset 1 covers 6 full squares ($6 \times 200\text{J} = 1200\text{J}$) and 5 partially filled squares. Inset 2 shows two of the partially filled squares. To improve accuracy, one could estimate the bottom square to be $3/4$ (0.75) full, and the upper square $1/5$ (0.2) full. This gives $(0.75 + 0.2) \times 200 \text{ J} = 0.95 \times 200\text{J} = 190\text{J}$ for these two squares. So this is how to do “counting squares”, full squares plus sum of the estimates of the partially full squares .

(2) For **cutting and weighing** one cuts out the unknown and a standard to the same scale from the same material and weigh them. In this example a cutout of the $200\text{N} \times 10\text{m} = 2000\text{J}$ rectangle serves as the standard. A cutout of the unknown using the same scale and material is used as the unknown. Since both items are from the same material, then they will have the same areal density ρ_{areal} , where



$$\rho_{\text{areal}} = \text{Weight} / \text{Area} .$$

Hence

$$\text{Weight}_{\text{rectangle}} / \text{Area}_{\text{rectangle}} = \text{Weight}_{\text{unknown}} / \text{Area}_{\text{unknown}} .$$

So,

$$\text{Area}_{\text{unknown}} = (\text{Weight}_{\text{unknown}} / \text{Weight}_{\text{rectangle}}) \text{Area}_{\text{rectangle}} .$$

4.3 Small Angle Approximation

Angle		Fractional Error		
θ	θ	$\sin\theta$	$\sin\theta/\theta$	$(\theta - \sin\theta)/\theta$
degrees	radians			
45	0.7853982	0.7071068	9.00316E-01	9.96837E-02
30	0.5235988	0.5	9.54930E-01	4.50703E-02
25	0.4363323	0.4226183	9.68570E-01	3.14303E-02
20	0.3490659	0.3420201	9.79816E-01	2.01845E-02
15	0.2617994	0.258819	9.88616E-01	1.13841E-02
10	0.1745329	0.1736482	9.94931E-01	5.06923E-03
9	0.1570796	0.1564345	9.95893E-01	4.10726E-03
8	0.1396263	0.1391731	9.96754E-01	3.24609E-03
7	0.122173	0.1218693	9.97514E-01	2.48585E-03
6	0.1047198	0.1045285	9.98173E-01	1.82670E-03
5	0.0872665	0.0871557	9.98731E-01	1.26876E-03
4	0.0698132	0.0697565	9.99188E-01	8.12115E-04
3	0.0523599	0.052336	9.99543E-01	4.56863E-04
2	0.0349066	0.0348995	9.99797E-01	2.03066E-04
1	0.0174533	0.0174524	9.99949E-01	5.07688E-05
0.1	0.0017453	0.0017453	9.99999E-01	5.07696E-07
0.01	0.0001745	0.0001745	1.00000E+00	5.07696E-09
0.001	1.745E-05	1.745E-05	1.00000E+00	5.07695E-11
0.0001	1.745E-06	1.745E-06	1.00000E+00	5.07639E-13
0.00001	1.745E-07	1.745E-07	1.00000E+00	5.00480E-15

It is well known that as the angle θ approaches 0, $\sin\theta$ approaches θ in value when θ is in radians. Symbolically

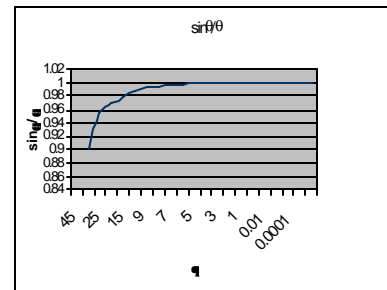
$$\sin \theta \rightarrow \theta (\text{radians}) \text{ as } \theta \rightarrow 0 ,$$

or

$$\lim_{q(\text{rad}) \rightarrow 0} \frac{\sin q}{q(\text{rad})} = 1$$

This is called the **small angle approximation**.

The table and graph below illustrate this property.



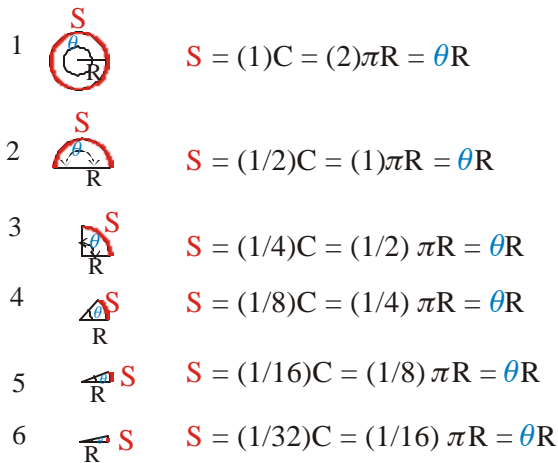
This is

very useful in many theoretical approximations like Simple Harmonic Motion (Sec 14).

4.4 Angles: Degrees and Radians

Most of us learn to express angles in the measure (or unit) of degrees, using symbol “°”. A full circle is 360°, half circle, 180°, a quarter circle, 90°, and so on. The degree is “man’s” unit for measuring angles. Its history goes back many thousands of years. Science and engineering use a different unit to measure angle when describing rotation like a turning wheel or fan blade, a spinning top and oscillators (like AC electric current, motion of a pendulum). They use a unit called the radian (abbreviated rad.). As we see in the following development, the radian might be called “nature’s” unit for measuring angle.

The picture below starts with a full circle of radius, **R**, then shows the sector perimeter, **S**, as the angle, θ (it is the Greek letter “theta”), decreases. The angle θ starts with a full circle and then shrink by 1/2 in each successive case. That is, 1, 1/2, 1/4, 1/8, 1/16, finally 1/32 of a circle. The figure shows the sector perimeter **S** the coefficient or multiplier of the circumference C and the coefficient or multiplier of πR . We immediately notice a one-to-one relation between the coefficients.



TABLE

Case	angle θ		
	$S=()$ C	$S=()$ πR	degrees
1	1	2	360°
2	1/2	1	180°
3	1/4	1/2	90°
4	1/8	1/4	45°
5	1/16	1/8	22.5°
6	1/32	1/16	11.25°

The TABLE lists the numerical values. The TABLE show $(2)\pi$ on the same line as 360°, $(1)\pi$ on the same line as 180°, $(1/2)\pi$ on the same line as 90°, and so on. So, we can conclude that

$$2\pi \text{ (some name)} = 360^\circ . \tag{1}$$

The name given to the 2π is radian, abbreviated rad. Equations 1 becomes

$$2\pi \text{ rad} = 360^\circ . \tag{2}$$

This tells us by dividing 360 by 2π , that

$$1 \text{ rad} = 360/2\pi^\circ = 57.2957759\dots^\circ \tag{3}$$

From this development we can say the radian is nature’s unit to measure angle.

A short table of degree <-> radian conversions

	angle										
degrees	0	30	45	60	90	120	135	150	180	270	360
radians	0.0000	0.5236	0.7854	1.0472	1.5708	2.0944	2.3562	2.6180	3.1416	4.7124	6.2832