7. Newton's Laws of Motion

Objectives

To learn via experiment and theory

1. Newton's Laws of Motion for both Linear and Rotational motions
2. the nature of Forces and Torques
3. the nature and roles of mass and moment of inertia.
4. to draw and use Free Body Diagrams (FBD).

THE EVENT

Hit the stick. Do ' Hit the Stick I' and 'Hit the Stick II', below - experience and discover force, torque, mass and moment of inertia (See Section 7.3).

Overview

Our human mechanical experiences primarily involve interactions associated with motion, more specifically, changing motion. In walking and running, we start from rest, speed up then slow down and stop. We trip and try to stop our fall. We hit and push to start or stop the motion of objects. We throw and catch. We are always experiencing net external forces and torques. When we are still, if we are comfortable then our thoughts turn to other things. If we are not comfortable, then our thoughts become concerned with what net external forces and torques we can apply to change the situation. Because of this we humans are very strongly Newton's 2nd Law creatures. Following this lead, here we will carry out the following scheme. First we will briefly state and discuss Newton's Laws of Motion in their usual order. Then we will cover each in detail in reverse order. To properly use Newton's 2nd law requires the 3rd Law.

7.1 Newton's Laws of Motion

7.1.1 Point Bodies and Extended Bodies.

We need to immediately distinguish between types of bodies.

1. the **point body**

   It has no spatial extent. Its length along all three spatial directions is zero (L=0, for all directions).

2. the **extended body**

   This has length greater than zero (L>0) along at least one spatial dimension . Then, there are two types of extended bodies,

   2a. **rigid bodies**

   These are defined as those bodies for which the relation between points on, or within, them are invariant. Such bodies cannot deform in any way - can not stretch, shear, or change shape or volume.

   2b. **non-rigid bodies**

   These bodies are able to deform.

7.1.2 Initial Statement of Newton's Laws of Motion.

Newton’s 1st Law of Motion:

A body subjected to no net external force maintains constant motion - it moves along a straight line at constant speed. This gives translational equilibrium, also called equilibrium of the 1st kind ;

$$\Sigma \mathbf{F} = \mathbf{0}.$$

An extended body subjected to no net external torque remains in constant rotational motion - it rotates constant angular velocity. This gives rotational equilibrium, also called equilibrium of the 2nd kind;

$$\Sigma \mathbf{t}_{\mathbf{o}} = 0.$$

Newton’s 2nd Law of Motion

For Translation: A body of mass m subjected to a net external force F experiences an acceleration given as

$$\Sigma \mathbf{F} = \mathbf{ma} \quad .$$

For Rotation: An extended body of Moment of Inertia I_o about the axis through O subjected to a net external torque about axis O experiences an angular acceleration given as

$$\Sigma \tau_o = I_o \boldsymbol{\alpha} \quad .$$

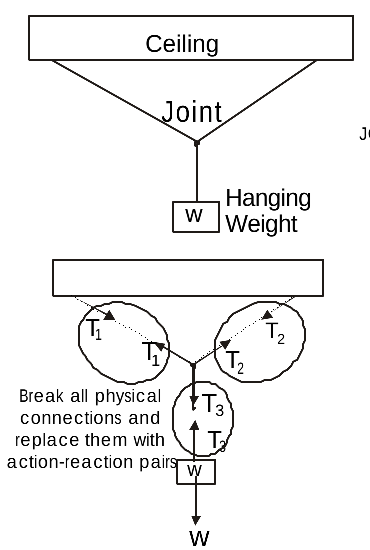
Newton’s 3rd Law of Motion:

For every action, there is an equal and opposite reaction.

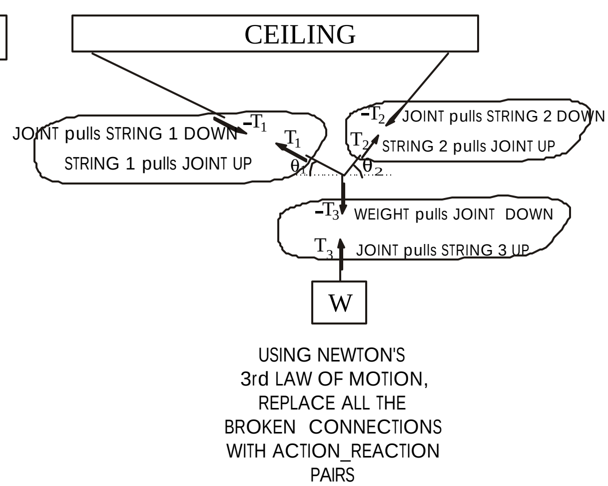
7.2 Newton’s 3rd Law of Motion:

For every action, there is an equal and opposite reaction.

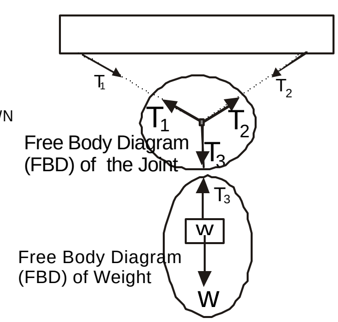
This allows us to draw free body diagrams of bodies which are critical for solving many problems. But, at first sight this is a mysterious statement. The mystery, however, clears under close examination. It is a statement of point of view - who sees what. The pictures below illustrate the meaning of this statement. The “Action-Reaction Pairs Annotated” are the key. Let’s look at the T_3's. The weights point of view is that the joint pulls it up. But, the joint’s point of view is that the weight is pulling it down. Similarly for the



Action-Reaction Pairs



Action-Reaction Pairs Annotated

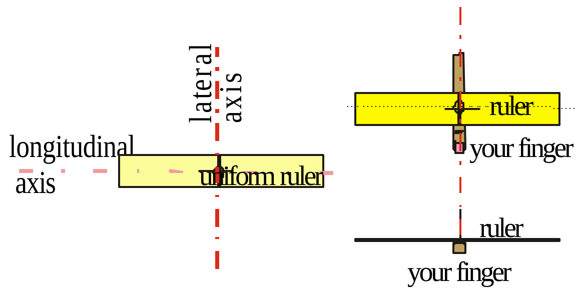


Free Body Diagrams

ceiling-joint views for the T_1 pair and the T_2 pair. In the middle picture the pairs are given their "true" names (e.g., T_1,-T_1). In the other diagrams the "-" is left out because the direction on the diagram takes care of the "-".

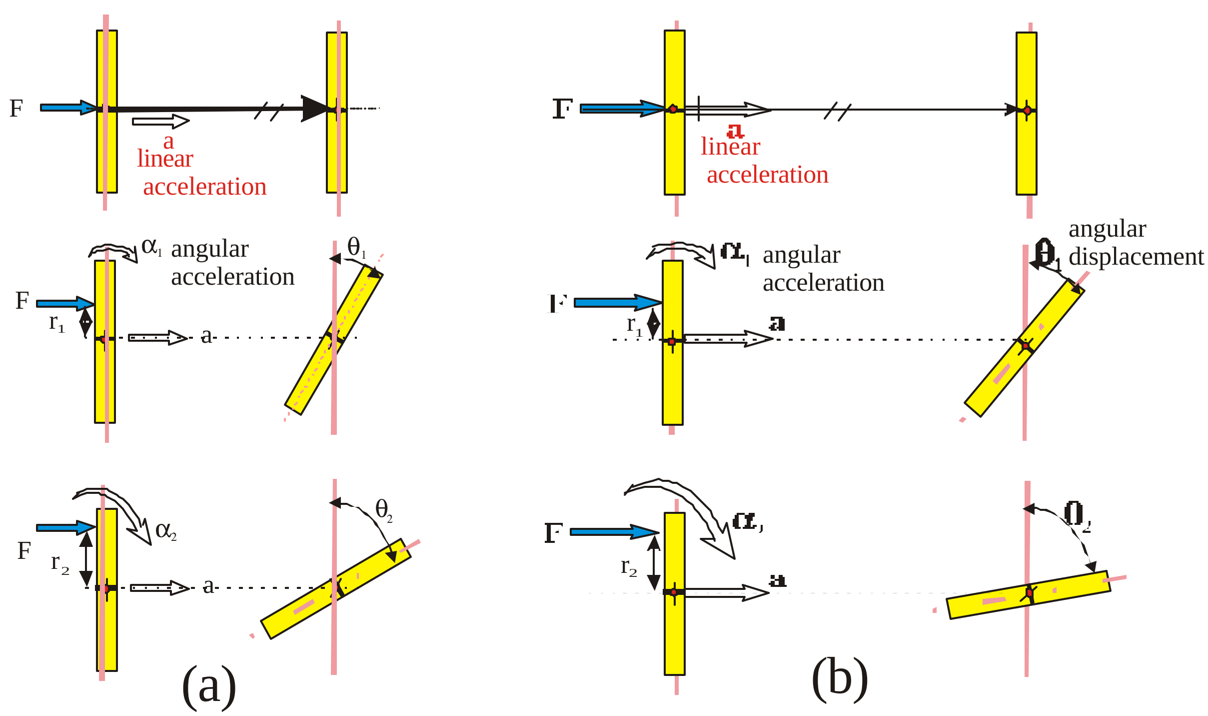
7.3 Newton's 2nd Law of Motion:

Get a smooth flat wood or plastic ruler that is uniform along its length. Uniform along its length means if you were cut it into many pieces parallel to the lateral axis, the pieces would all look alike. Find it and mark the lateral line where the ruler balances on your finger as shown in the picture to the left. Later we will find that this balance point is called the longitudinal center of mass.



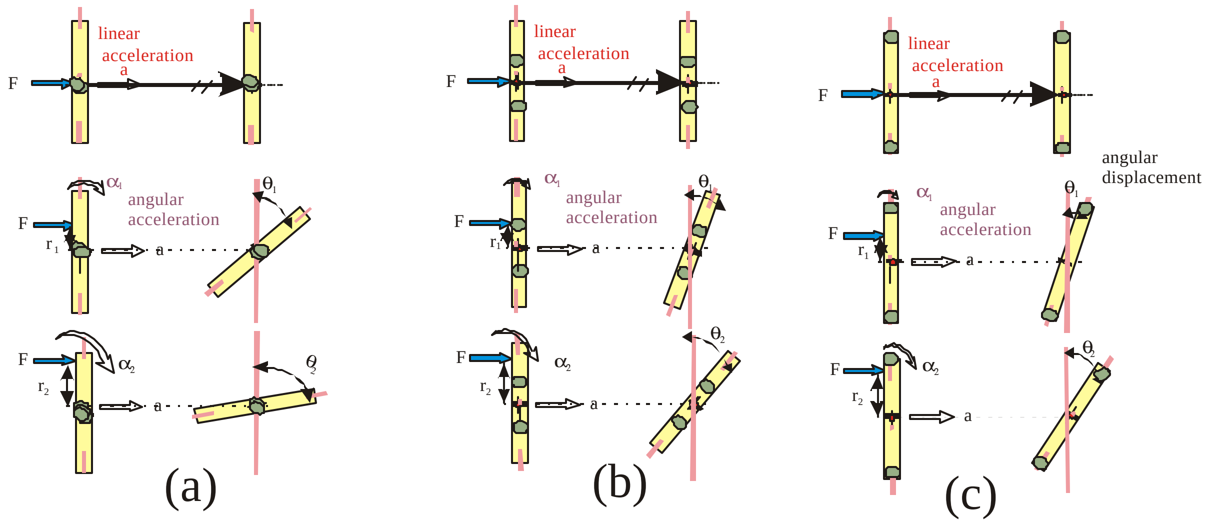
Then lay it on a smooth flat level surface, like a very, very, very clean wood, plastic or glass table top, desk top or floor. It is very important to limit the effects of friction as much as possible. Clean all surfaces with ammonia, rather than soap. Soap leaves a residue that can be hard to get off. Then "polish" with a dry clean cloth or handy wipe. With your finger, hit the ruler at its balance point with a quick, sharp blow. Then hit it again with the same force but at two different distances, r_1 and r_2 , from the balance point. Notice the motion of the ruler as it moves along the smooth flat level surface. The figures (a) and (b), below, illustrate the situation. The right, Fig. (b), has a larger force, but the same two values of r_1 and r_2 .

Hit the Stick I



Hit the Stick II

Now let's take two equal masses of clay and stick them on the ruler and hit the ruler again as shown below.



7.3.1 Observations and Reflections:

We observe two major effects. First, the force acting parallel to the lateral axis causes the ruler to move along a straight line parallel to the lateral axis - translational motion. When the force acts at the center of mass, this is the only effect. Secondly, when the force acts at a longitudinal distance, a displacement perpendicular to the force, from the center of mass, the ruler has the same translational motion, but now it also rotates . As this longitudinal distance, displacement perpendicular to the force, increases ($r_2 > r_1$), rotational motion increases, but the translational motion stays the same. This new entity that causes the rotation is called a torque.(Section 7.3.3)

What did **you** observe?

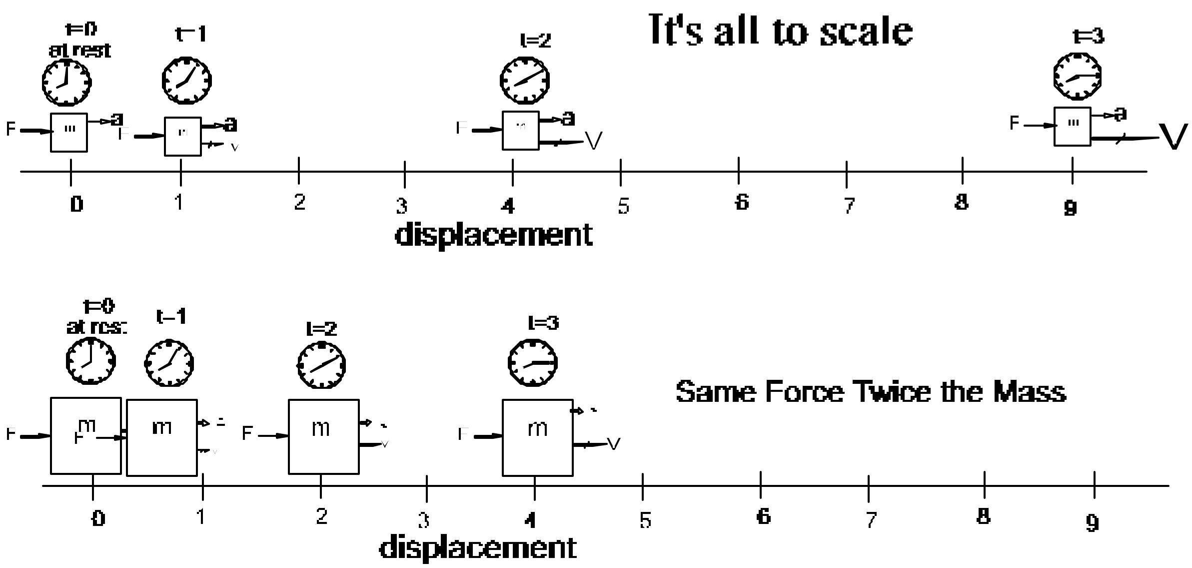
1. What type of motion did forces affect?
2. What type of motion did torques affect?
3. What happens when the fixed force remains fixed as the mass deceases? Increases?
4. What happens as the force on a fixed mass increases? Decreases?
5. What happens when the force and the body remain fixed as the displacement of its application , r, increase? Decreases?
6. What happens when the torque and the mass of the body remains fixed while the clay pieces are moved to greater displacement from the center?

But what are these two physical entities? What are they? What do they really do? We will discover that forces are simple entities. They are a push or a pull that can be reduced to a 1-dimensional vector. Torque, on the other hand, are much more complicated in that they require full 3-dimensions to describe or represent them. Let's explore these issues, first for forces, then for torques.

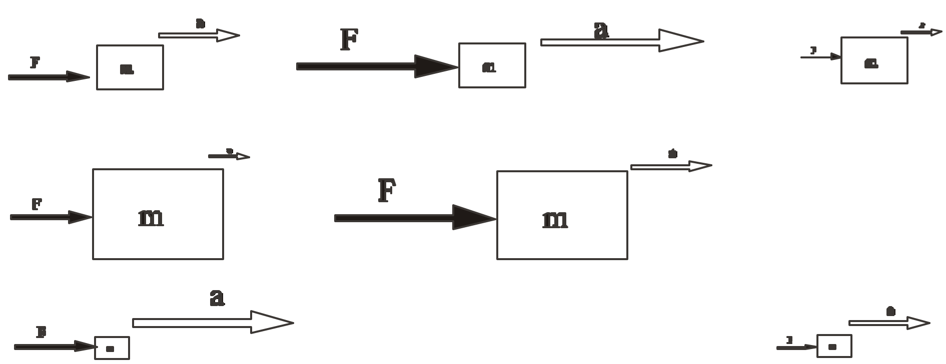
7.3.2 Forces and Newton's 2nd Law for Translational :

A body of mass m subjected to a net external force F experiences an acceleration **a** given as

$$\bar{F} = m\bar{a}$$

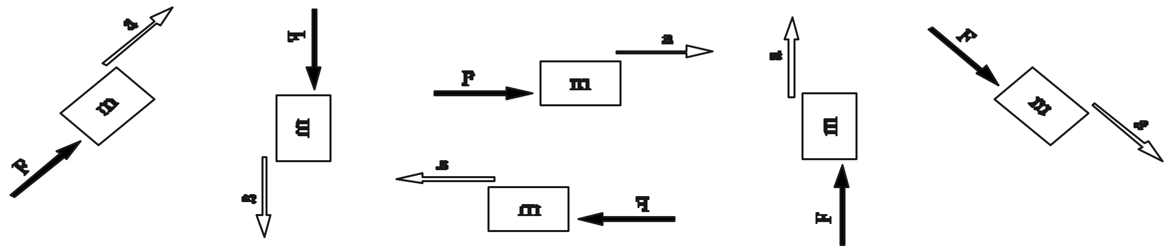


The above figure depicts the behavior of two bodies of different masses subjected to the same net external force. They start from rest, $v_o = 0$. The bodies are shown at their respective displacements at equal time intervals. The clock above the body at each position registers the time. The force, acceleration and velocity vectors are on each the body at each position. Everything, displacements and vectors, is to scale. Carefully note the sizes of the displacement, force , acceleration and velocity at each position.



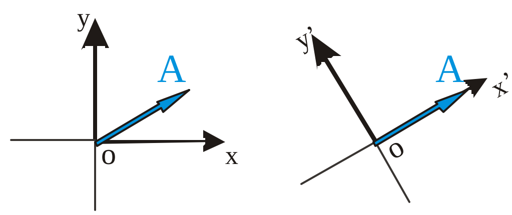
The above picture graphically illustrates the **F**=m**a** relationship. If either the resisting mass (inertia) m -> m/2, or the causing forces **F** -> 2**F**, the resulting response, the acceleration**, a** doubles, and so on.

What about when m-> 2m, or **F** -> **F**/2 ?



The above picture graphically illustrates the directional nature of **F**=m**a**. Note the directional relationship between the causing force **F** and the resulting response, the acceleration, **a**

A force, **F**, is simply a push or a pull. It is directional, so it is a vector. It affects translational (linear) motion. All linear motion can be reduced to motion on a one-dimensional straight or curved line. Some curved lines, as you will learn in your later travels in calculus, can be represented in curvilinear coordinate systems. For example, on the spherical earth, a line of constant longitude, or any circle on the sphere that has the same radius as the sphere, is a geodesic "straight line".



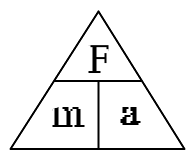
Forces in Euclidean system can be represented as a one-dimensional vector by using an appropriate coordinate system. In the picture, the vector **A** is 2-D in the x-y coordinate system, but 1-D in the x'-y' system.

Inertia is the tendency of a body to resist change in its linear motion. The **mass**, given the symbol m, is a measure of inertia. Mass has no directional effects. It is a scalar. Mass is a fundamental dimension.

7.3.2.1 Memory Aids

The Triangle

F = ma , a = F/m , m = F/a .



A mnemonic F = ma -----> Fma (say it)!

7.3.2.2 Cause and Effect

In the Newtonian view, whenever there is an acceleration there must be a net external force F_{ext} causing it. That is, forces cause accelerations. Acceleration is a response to a force. Accelerations do not cause forces!!!

7.3.2.3 Examples:

1. A body of mass m = 4 kg experiencing a net external force **F** = 24 N East. What is its acceleration?

F = m**a**. So, **a** = **F**/m = 24 N East / 4 kg = (24/6) m/s^2 East = 4 m/s^2 East

2 A body of mass m = 5 kg is undergoing an acceleration **a** = 10 m/s^2 45° South of East. What is the causing force **F** ?

F=m**a**, **F** = 5 kg x 10 m/s^2 45° South of East = (5x10) (kg m/s^2) 45° South of East = 50 N 45° South of East.

7.3.2.4 Exercise

1. Get several objects of widely varying masses (observed by way of their weight). Place them on a smooth table or floor. Using about the same force, hit each object in many different directions. Record the "amount of force" and its direction, the mass of the object and its response for each hit.

Levels of Forces: A very small force might be one where only the fingers of the hand move as hard as you can - the hand itself does not move at the wrist. A larger force might be a wrist slap - the whole hand moves as hard as you can at the wrist. The next level of force might be where the forearm moves at the elbow as hard as you can - no movement of the upper arm except for rotation. In each case "as hard as you can" will be more reproducible than to do it "softly", then a little harder" , and so on. Also show a pictures of some of your examples.

Sample Data Table

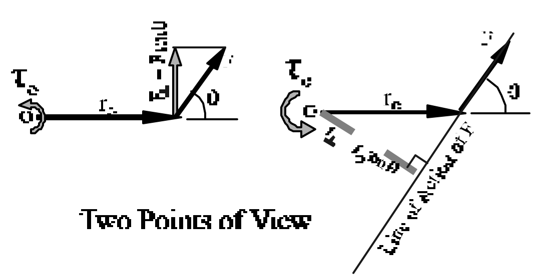
Object and its mass	Force		Response	
	Magnitude	Direction	Magnitude	Direction
can of soup 16	*Hard*	*to the left*	*quickly moved 12"*	*to the left*

2. Calculate the acceleration a when a body of mass m = 2 kg is subjected to a force F = 10 N 45° N of E.

7.3.3 Torques and Newton's 2nd Law for Rotation

7.3.3.1 Torques about the axis through the point o , $\mathbf{t}_o$

Torque causes changes in rotational motion. As we have seen in the previous chapter on motion, rotational motion is a full three-dimensional entity. When the observed motion is in a plane, the motion is referenced as being around an axis perpendicular to that plane. Hence rotational motion is described by the vector or cross product. Torque causes the fully 3 dimensional rotational motion described by the vector product, then torque itself is full 3-Dimensional entity
described by the vector product. It is defined as follows.



$$\mathbf{t}_o = \mathbf{r_o} \text{ x } \mathbf{F}$$

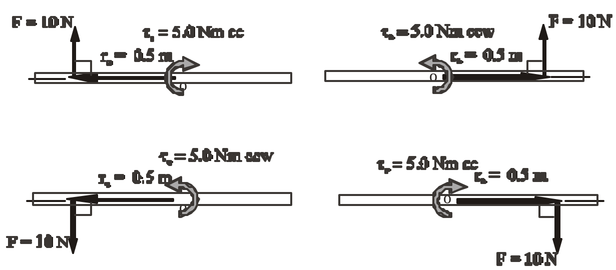
$$*\mathbf{t}_o* = \mathbf{t}_o = r(F\sin\theta) = rF_{\perp},$$

perpendicular force $F_{\perp} = F\sin\theta$.

$= (r\sin\theta)\, F = r_{\perp} F$,

perpendicular distance $r_{\perp} = r\sin\theta$,

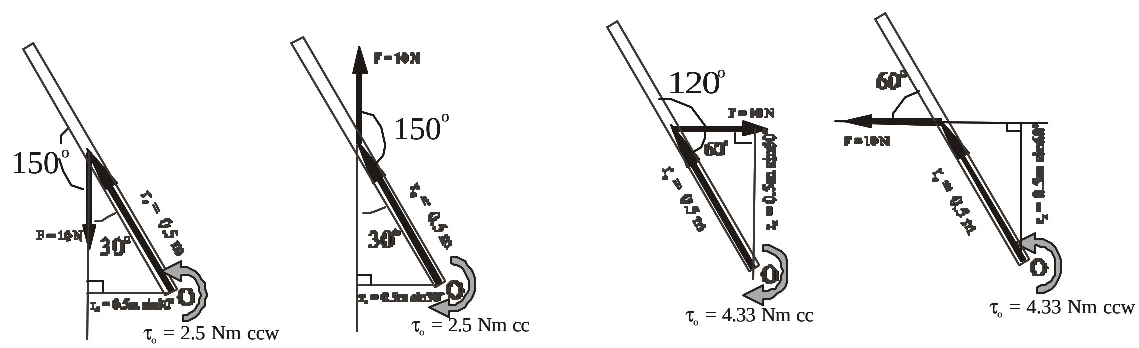
The torque vector, $\mathbf{t}_o$, is perpendicular to the plane of the paper and passes though point o.
$F_{\perp} = F\sin\theta$ is the component of the force that is perpendicular to the vector $r_{o.}$
$r_{\perp} = r\sin\theta$ is the shortest and perpendicular distance from the line of action of the force F to the point O



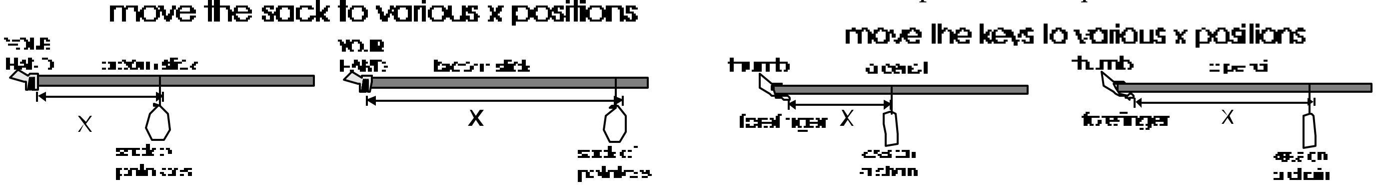
The point O is often called the **fulcrum** .

The distance r_o is often called the **lever arm** or the **moment arm**.

Torques about the axis O can tend make a body rotate about that axis clockwise (cw) or counter clockwise (ccw). Some examples follow.



7.3.3.2 Exercise: Experience Torque



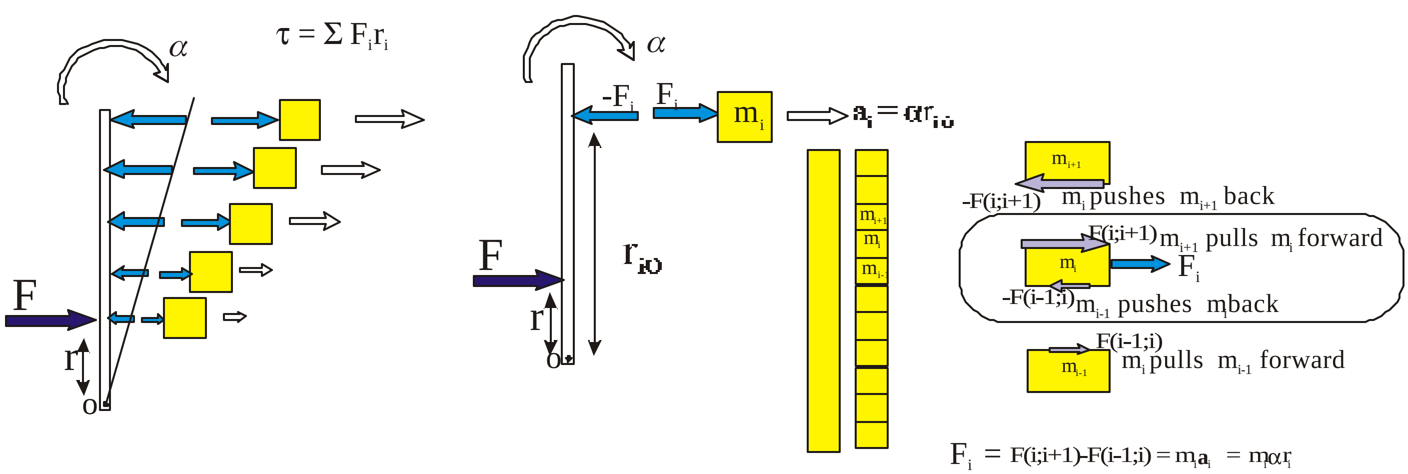
As you move the load, the sack or keys, record the distance x and your observations - what do you feel. How hard or easy is it for you to support this situation. Use several "sacks" of different masses.

Sample Data Table

Load - Item	Load Weight	x	Torque	your experience
Potato Sack	5 p	1 ft	5p x 1f = 5 pf	easy to support
Potato sack	5 p	3 ft	5p x 3f = 15 pf	almost broke my wrist - couldn't handle it!

7.3.3.3 Newton's 2nd Law for Rotation

$$S\, t_O = I_O\, a$$



Let's imagine a rod as shown above. We can break the rod into mass elements m_i . The mass elements interact through interatomic forces as illustrated in the encircled inset. Now let's look at the top mass element m_i that is a distance r_{iO} above the point of rotation O. This element is acted on by the net effect force F_i and undergoes the response acceleration $a_i = \alpha r_{iO}$, where α is the angular acceleration of the whole rod. **<u>In a rigid body every piece of the body has the same angular motion</u>** **(See Sec. 7.1.1)**. So, the mass elements m_i is acted on by the torque τ_{io} about the point O given by

$$\tau_{io} = F_i\, r_{io}\ .$$

Using Newton's 2nd Law for Translation F = ma , this becomes

$$\tau_{io} = m_i\, a_i\ \ r_{io}\ .$$

Since the relationship between linear acceleration a and angular acceleration α is $a = \alpha r$, we get

$$\tau_{io} = m_i\, \alpha\, r_{io}\ \ r_{io} = m_i\, r_{io}^2\, \alpha\ .$$

Summing over all of the mass elements gives

$$\Sigma\, \tau_{io} = \Sigma\, m_i\, r_{io}^2\, \alpha = (\, \Sigma\, m_i\, r_{io}^2)\, \alpha\ .$$

By Newton's 3rd Law, this sum of internal torques must equal the net external torque $\Sigma\tau_{ext} = \mathbf{Fxr}_o$. So,

$$\Sigma\tau_{ext} = (\, \Sigma\, m_i\, r_{io}^2)\, \alpha = I_o\, \alpha,$$

where I_O is called the moment of inertia about the point O, or about the axis perpendicular to the plane through O, and is defined as

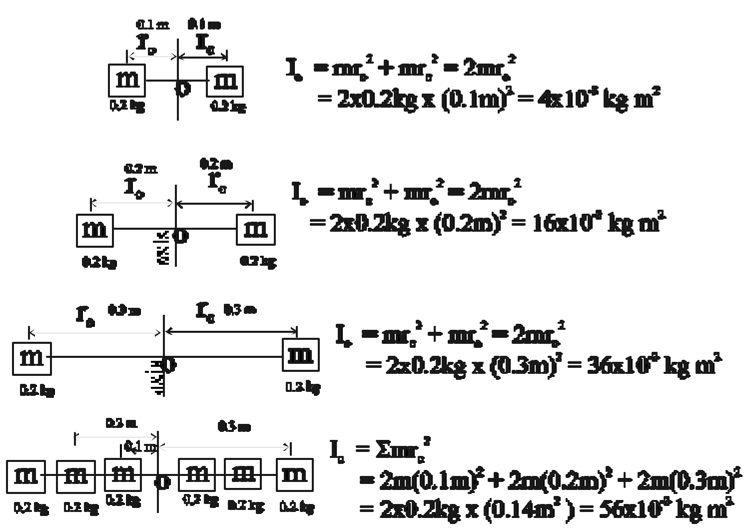
$$I_o = \Sigma\, m_i\, r_{io}^2\ .$$

Note that I_O depends on the masses and their distribution. Now let's look at the moment of inertia.

7.4 Moment of Inertia about o. $I_o = S\, m_i r_{io}^2$. [ML^2]

I_O depends the masses and their distribution about the axis through O. It has the physical dimensions [ML^2] and IS unit kg m^2 . Now let's look at some very simple examples.

7.4.1 Examples:



7.4 2 Exercises:

1. Verify the above values for I_o.
2. Show that if r doubles then I_o increases by four times. If r triples, I_o becomes 9 times larger.

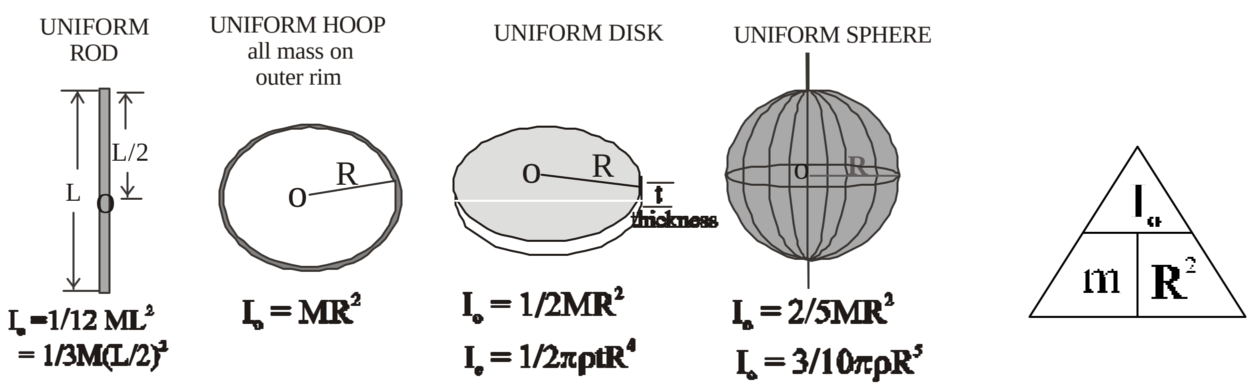
7.4.3 Experience Moment of Inertia I_o

1. Unscrew a broom stick from a broom.
2. Find the center of mass of the stick by finding the point at which it balances.
3. Mark this point. Mark off 15 cm, 30 cm and 45 cm on both sides of the stick.
4. Tie equal weights at the two 15 cm marks one on each side of the center. The weights might be two 5 p sack of potatoes, two 1 p boxes of sugar.
5. Hold the loaded stick try gently rotating by a few degrees. Record your feelings.
6. Move the weights to the two 30 cm marks. Do 5 again.
7. Move the weights to the two 45 cm marks. Do 5 again.
8. Use two new weights that are equal to each other but different from the first set. Do 5 to 7 again.

Sample Data Table

Item and weight	position	$I_o = \Sigma m_i r_{io}^2$	comment
potato sack 5 p = 2.27 kg	30 cm =0.3m	$2x(2.27kgx(0.3m)^2 =$ 0.409 kg m^2	very hard to rotate
sugar 1 p= 0.453 kg	15 cm = 0.15m	$2x(0.453kgx(0.15m)^2$ = 0.0204 kg m^2	very, very easy to rotate

7.4.4 Moments of Inertia of some common bodies about their center of mass



In the above picture the mass M and moment of inertia I_O They are all given in terms of their mass and effective radius, R, and L/2 for the rod. I_o of the disk and sphere are given in terms of mass density ρ. For generalization, I_o can be written as $I_o = \beta MR^2$, where β is a mass distribution parameter. The value of β approaches 0 as the mass becomes centered at the axis O. The maximum value is $\beta = 1$. The values of β for the above bodies are:

β = 1/3 for rod when L/2 is treated as R, β = 1 for hoop,
β = ½ for disk, and β = 2/5 for sphere.

Moment of Inertia

R or L/2 =	1.00 ' m			2.00 ' m			3.00 ' m		
M =	0.10 ' kg	1.00 ' kg	10.00 ' kg	0.10 ' kg	1.00 ' kg	10.00 ' kg	0.10 ' kg	1.00 ' kg	10.00 ' kg
Rod	0.033 ' kg*m^2	0.33	3.33	0.13	1.33	13.33	0.30	3.00	30.00
Sphere	0.040 ' kg*m^2	0.40	4.00	0.16	1.60	16.00	0.36	3.60	36.00
Disk	0.050 ' kg*m^2	0.50	5.00	0.20	2.00	20.00	0.45	4.50	45.00
Hoop	0.100 ' kg*m^2	1.00	10.00	0.40	4.00	40.00	0.90	9.00	90.00

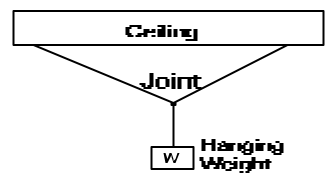
7.4.4.1 Exercises:

1. Verify in detail, including units, values of I_o in the above table.
2. Show that for a disk of the same material if the radius R is reduced to ½ of its original value that the moment of inertia I_O becomes 1/8 of its original value. If R is reduce to 1/3 it original value, I_o becomes 1/27 as large. How and why might this be of interest in hard drives in portable computers?

7.5 1st Law:

A body subjected to no net external force maintains a constant motions - moves along a straight line at constant speed. This gives translational equilibrium, also called **equilibrium of the 1st kind** ;

$$\Sigma \mathbf{F} = \mathbf{0}.$$



Equilibrium of the 1st Kind

A body subjected to no net external torques remains in constant rotational motion - moves with constant angular velocity. This gives rotational equilibrium, also called **equilibrium of the 2nd kind**;

$$\Sigma \mathbf{t}_0 = 0.$$

We can only discover the meaning of these ideas through Newton's 2nd Law of Motion Further, we can solve only the simplest definitional problems without Newton's 3rd Law of Motion. So we must make a journey through all three laws before we can make practical uses of forces and torques.

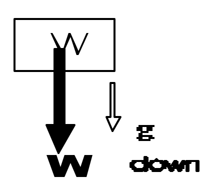
7.6 Some Definitions and Notions

Force F [ML/T^2] - is a push or a pull. It's a vector. The dimensions ML/T^2 follows from **F**=m**a**.

Weight W is the force due to gravity and always acts downward. It defines down.

$$\mathbf{W} = m\mathbf{g},$$

where **g** is the downward acceleration due to gravity. **g** = 32 f/s^2 = 9.8 m/s^2 on the surface of the earth.



Newton <-> Pound Conversion

W = mg ; So 1kg x 9.8 m/s^2 = 9.81 N. Thus 1 kg weighs 9.8 N.
But, on earth, 1 kg mass is **measured** to weigh 2.206 p.

$$1N = 1N(\frac{1}{9.81})\frac{kg}{N}(2.06)\frac{p}{kg} = 0.2248\,p, \quad \text{thus} \qquad 1\,p = 4.447\,N.$$

7.7.1 Problem Newton's Laws of Motion for Translation. The problem on the left is for static equilibrium. The one on the right is the same system but now in an accelerating elevator. Carefully notice the very small, but physically very significant differences between the two.

State the Principles	**Fill in the Details**
$+\wedge\Sigma F_v=0$;	$T_1\sin\theta_1 + T_2\sin\theta_2 - T_3 = 0$
vertical direction	
$+>\Sigma F_h = 0$;	$-T_1\cos\theta_1 + T_2\cos\theta_2 = 0$
horizontal direction	
$+\wedge\Sigma F_v = 0$;	$T_3 - mg = 0$
vertical direction	

After some algebra we get

$T_1 = mgcos2_2/\sin(\theta_1+\theta_2)$

$T_2 = mgcos2_1/\sin(\theta_1+\theta_2)$.

State the Principles	**Fill in the Details**
$+\wedge\Sigma F_v = \mathbf{m_{joint}\ a}$;	$T_1\sin\theta_1 + T_2\sin\theta_2 - T_3 = 0$
	$m_{joint}a = 0$ because $m_{joint} = 0$.
$+>\Sigma F_h = 0$;	$-T_1\cos\theta_1 + T_2\cos\theta_2 = 0$
no acceleration in the horizontal direction	
$+\wedge\Sigma F_v = \mathbf{ma}$;	$T_3 - mg = \mathbf{ma}$
vertical direction	

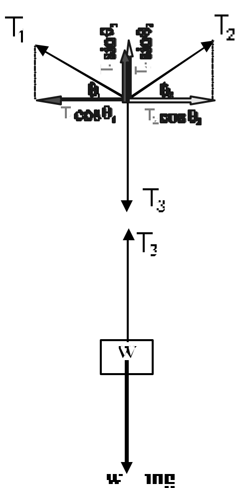
After some algebra we get

$T_1 = m(g+\mathbf{a})\cos\theta_2/\sin(\theta_1+\theta_2)$

$T_2 = m(g+\mathbf{a})\cos\theta_1/\sin(\theta_1+\theta_2)$.

NOTES:

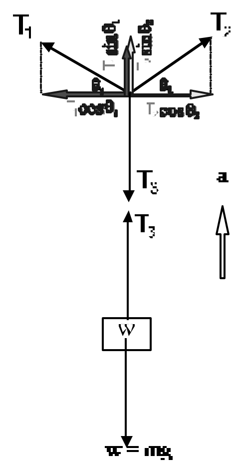
1. Notice the layout. Next to each Free Body Diagram(FBD) , State the Principles, then fill in the details appropriate to that body. This layout helps you to keep track of things. For each force on the FBD there should be one entry in the details



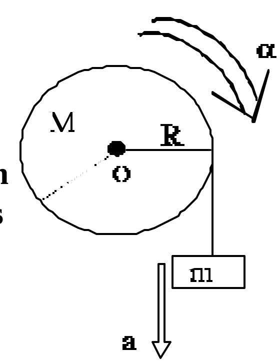
NOTES- Continued

2. Always be sure to put your assumed + direction in the Principles and stick to it in the Details.

3. The grey writing is the same for both.. Notice that the only difference between the two problems is the acceleration of the elevator, **a** .



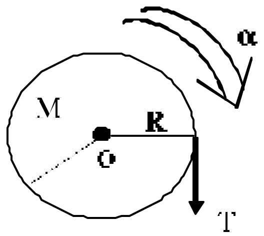
7.7.2 Problem Newton's Laws of Motion for Rotation



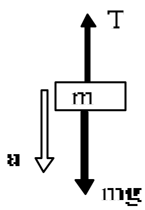
Thread is wrapped around a the rim of a spool. The spool has mass M, radius R and moment of inertia about O, I_O = $MR^2, where $ is a constant. The axis of rotation, O, is fixed. A mass m is attached to the end of the thread as shown in the figure. Find the angular acceleration, ", in terms of M, m and R. The thread does not slip.

a.

Draw the Free Body Diagrams of the wheel and the mass. *30 points* | State the Physical Principles *10 points* | Fill in the Details *30 points*



$\Sigma\tau_O = I_O\alpha$; TR = $\beta MR^2\alpha$, (1)



+down$\Sigma F = ma$; mg - T = ma , (2)

no slipping a = Rα . (3)

<u>THE PHYSICS IS DONE!</u>

THE MATHEMATICS

b. Find " in terms of m, M, R and g. *30 points*

Dividing (1) by R gives T = $\beta MR\alpha$. (4)

Rewriting (2) gives mg - T = ma . (5)

Add (4) and (5) to eliminate T gives : mg = MRα + ma .

Now substitute (3) in the above gives, mg = $\beta MR\alpha + mR\alpha = (\beta M+m)R\,\alpha$.

So, $$a = \frac{mg}{(b\,M+m)R} = \frac{g}{(1+b\,(M/m))R} \quad (6)$$

dividingthroughbym

c. Find the tension in the thread T and the linear acceleration of mass m.

From (4) we get T as $$\mathrm{T} = b\,MRa \;=\; b\,(M/m)\frac{mg}{(1+(b\,M)/m)} \quad (7)$$

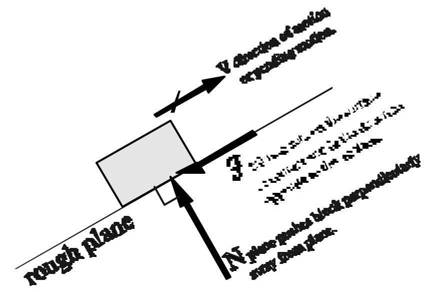
From (3) we get a as $$a = Ra = \frac{g}{(1+b(M/m))} \quad (8)$$

Discuss the behavior of α, a and T as β and [M/m] vary .

8. CONTACT (SURFACE) FRICTION

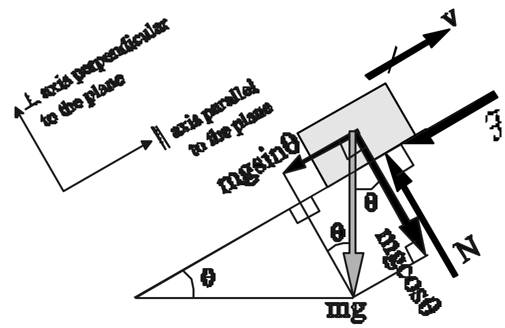
8.1 Contact (Surface) Friction

8.1 Contact (Surface) Friction is a force between two surfaces. As with all friction, it always acts opposite to the direction of actual or impending motion, opposite to **v**. See **Dissipative Forces** below.



Static Friction, $\mathbf{F}_s$ is the friction force acting before motion starts. Its value is equal to or less than the opposing active forces and $0 < \mathbf{F}_s < \mu_s N$, where μ_s is called the coefficient of static friction. N is the normal force, it's perpendicular to the surfaces in contact.

Kinetic Friction, $\mathbf{F}_k$, takes over once the surface move relative to each other. $\mathbf{F}_k = \mu_k N$, where μ_k is the coefficient of kinetic fiction. $\mu_k < \mu_s$.



Some Approximate values

MATERIALS	static μ_s	kinetic μ_k
steel on steel	0.74	0.57
brass on steel	0.51	0.44
glass on glass	0.94	0.40
copper on glass	0.68	0.53
teflon on teflon	0.04	0.04
rubber on dry concrete	1.00	0.80
rubber on wet concrete	0.30	0.25

The **NORMAL** is that force which is **always perpendicular** to the surfaces at the point of contact.

8.2 Exercises:

8.2.1 Show that the angle of the inclined plane θ is also the angle between the weight mg and its component (mgcosθ) perpendicular to the inclined plane as shown in the diagram.

8.2.2. **DO THE FRICTION SLIDE** (It's corny but it makes a point)

1. First, stand and put your feet together. Lift your weight off your right foot and slide it to the front. Then lift your weight off your left foot and slide it to your right foot. Weight off your right foot and slide it to the back. Weight off your left foot and slide it to your right foot.
2. Now put a little weight on your right foot and slide it to the front. A little weight on your left foot, slide to your right foot. Then slide back as before with a little weight on the sliding foot.
3. Do it all again with more weight on the sliding foot.
4. Finally, with as much weight as you can put on t the sliding foot, slide, slide , slide.

WHAT DO YOU OBSERVE?
Use as many **foot-floor** variations as you can.

Foot - Bare, cotton sock, nylon sock, wool sock, leather soled shoes, rubber soles shoes (sneakers)
Floor - Smooth wood, vinyl tile, linoleum, ceramic tile (often in the bathroom), bath tube or shower floor.

SAMPLE DATA TABLE

Foot	Floor	Weight on sliding foot	Comments
in cotton sock	ceramic tile	light	slides very ,very easily
in cotton sock	ceramic	very heavy	slides with some difficulty

ANOTHER FORM OF DATA TABLE

Combinations		Weight on sliding foot			Sliding the foot			
Foot	Floor	light	moderate	heavy	easy	not easy	hard	didn't
sneaker	tile		x				x	
sneaker	tile			x				x

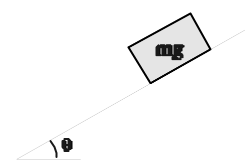
8.2 LAWS OF CONTACT FRICTION

$$F_s =< F_{s\,MAX} = \mu_s N > F_k = \mu_k N$$

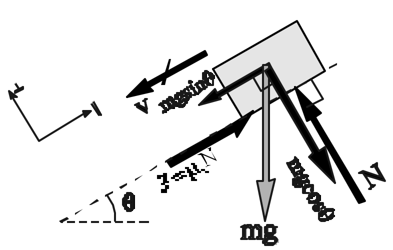
These are empirical laws because they come directly from experience and experiment. We do not know why they behave his way. The answers to that question is in the hands of theorist. So far the explanations are very complex, complicated and incomplete.

8.2.1 Example: Finding m_s

A block is at rest on a rough variable angle inclined plane. At some critical angle, $\theta_{critical}$, the block just begins to slide down the plane. Show that the coefficient of static friction $\mu_s = \tan\theta_{critical}$. This angle is often called the angle of repose.



Draw a Free Body Diagram of the Block just as it start to slide



State the Physical Principles

Laws of friction

$Fs >= F_{s\,MAX} = \mu_s N$

Newton's Laws of Motion

$$+\swarrow\ \Sigma F_{||} = m_1 a_{||}\ ;$$

sum of forces parallel to the plane =0. Down plane is +

$$+\nwarrow\ \Sigma F_{\perp} = 0;$$

Fill in the Details

Here we are concerned with $F_{s\,MAX}$ because it is just at the onset of moving. So,

$Fs = F_{s\,MAX} = \mu_s N$ (1)

$mg\sin\theta_{critical} - \mu_s N = 0$ (2)

sum of forces perpendicular to the plane =0. Upward is + direction. $N - mg\cos\theta_{critical} = 0$ (3)

Eq (2) gives $N = mg\cos\theta_{critical}$. (4)

Substituting this into Eq(3) yields $mg\sin\theta - \mu_s\, mg\cos\theta_{critical} = 0$. (5)

Solving Eq(5) and canceling mg gives $\sin\theta_{critical} = \mu_s \cos\theta_{critical}$ (6)

Dividing both sides by cos2 and using the definition tan2 = sinθ/cosθ, it follows that

$$\mu_s = \sin\theta_{critical} / \cos\theta_{critical} = \tan\theta_{critical} .$$

8.2.2 Exercise: Measure μ_s

Get a hard cover book , a protractor , some objects like coins, a tape cassette case, pieces of various kinds of paper and cloth materials , for each one piece the size to wrap around , tape to, or glue to, the tape cassette case and another piece large enough to cover the surface of the book..

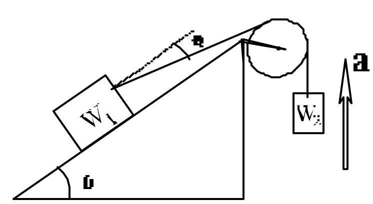
1. Place a coin on the book while it is in a horizontal position. Then raise the angle of the book until the coin just starts to slide. Measure this angle with the protractor and record it. The tangent of this angle is the coefficient of static friction.
2. Next, cover the surface of the book with a piece of smooth notebook paper. Measure and record the above two angles for the sliding coin on this paper.
3. Next, cover the surface of the book with a piece of newspaper. Measure and record the above two angles for the sliding coin on this paper.
4. Next, cover the surface of the book with a piece of glossy paper. Measure and record the above two angles for the sliding coin on this paper.
5. Next, cover the surface of the book with a piece of cloth. Measure and record the above two angles for the sliding coin on this surface.
6. Now using a tape cassette case, first bare, then covered with the smooth notebook paper, then with newspaper, then with the glossy paper, with the cloth, measure and record the two angles for these sliding down all of the previous surfaces on the book.

You may not have time for all of these variations. But, do at least a dozen variations once each. Also do at least one ten times and calculate the average and standard deviation.

SAMPLE DATA TABLE

Item	Surface	Angle Start	μ_s	Angle Stop	μ_k
quarter(clean)	newspaper	24^o	0.445	18^o	0.325
quarter (clean)	notebook cover	31^o	0.601	23^o	0.424

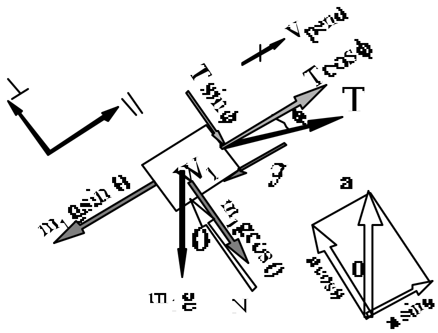
8.3 Example: Connecting cord NOT parallel to the inclined plane



The system as shown has $W_1 = m_1g$ on a plane inclined at an angle θ to the horizontal. It is connected to weight $W_2 = m_2g$ by a massless inextensible rope going over a massless frictionless pulley. The rope at m_1g makes an angle ϕ with the plane. The coefficients of static and kinetic friction between m_1g and the plane are μ_s and μ_k, respectively. **The whole system has an upward acceleration a.** Show that the normal, N, acting between the plane and m_1g is

$$N = m_1(g+a)\cos\theta + m_2(g+a)\sin\phi .$$

a. Sketch the Free Body of Diagram of W1 — 20 points | State the Physical Principle(s) — 20 points | Fill in the Details For each — 30 points



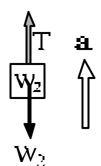
Laws of Friction $F_s =< F_{smax} = \mu_s N > F_k = \mu_k N$ (1)

Express in terms of components perpendicular and parallel to the plane. The action is along these directions.

$+\nwarrow \Sigma F_\perp = m_1 a_\perp$; $N - m_1g\cos\theta - T\sin\phi = \mathbf{m_1 a \cos\theta}$ (2)

$+\nearrow \Sigma F_{||} = m_1 a_{||}$; $-F - W_1\sin\theta + T\cos\phi = \mathbf{m_1 a \sin\theta}$ (3)

Sketch the Free Body Diagram (FBD) of W_2. 10 points



The type of friction is left open here. If we are concerned with the onset of motion then we use $F_{smax} = \mu_s N$, *and* $F_k = \mu_k N$ *for kinetic*.

$\Sigma F_h = 0$; *Trivial because there are no horizontal forces.*

$+\wedge\Sigma F_v = \mathbf{m_2\, a}$; $T - m_2 g = \mathbf{-m_2\, a}$ (4)

b. From the above, solve for the expression for the normal acting between W_1 and the plane . 10 points

Equation (4) gives us the value of T *as* $T = m_2(g + \mathbf{a})$.

Substituting this into Equation (2) give

$$N = m_1 g\cos 2 + T\sin N + \mathbf{m_1 a \cos\theta}$$
$$= m_1 g\cos\theta + m_2 \sin\phi(g + \mathbf{a}) + \mathbf{m_1 a \cos\theta}$$
$$= (g + \mathbf{a})\, m_1\cos\theta + (g + \mathbf{a})\, m_2 \sin\phi .$$

c. Briefly discuss why the normal behaves this way. 10 points

The value of the normal depends on all forces perpendicular to the plane. The tension T has a component perpendicular to the plane

9 Centripetal Force

Review Section 6 (Centripetal Acceleration) all !!!

In Newtonian Physics whenever there is an acceleration, there must be a causing force - Newton's 2nd Law of Motion,

$$a = F/m.$$

That is, the acceleration is the response to a force - an acceleration implies there must be a force. So the force causing the centripetal acceleration a_c is called the centripetal force F_c and when acting on a body of mass m has the value

$F_c = ma_c = mv^2/R = mT^2R.$

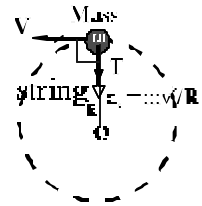
9.1 Examples:

9.1.1 A mass swinging around on a string - such as swinging a set of keys on a key chain.

$$\Sigma F = ma = ma_c = mv^2/R$$

$F = T$ (Tension in the string)

So, $T = mv^2/R$



9.1.2 A satellite orbiting a parent body -such as the moon around the earth, any planet around the sun

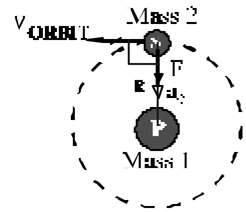
$$\Sigma F = ma = ma_c = mv^2/R$$

$$F = F_{Gravity} \sim \frac{\text{Mass 1 x Mass2}}{R^2}$$

$v = v_{ORBIT}$

So, $m_{Mass2} v_{ORBIT}^2/R \sim$ Mass1 x Mass2 $/R^2$

Hence, $v_{ORBIT} \sim$ [Mass1 x Mass2 $/(m_{Mass2}R)]^{1/2}$



9.1.3 A negative charge orbiting a positive charge - such as an electron orbiting a proton in the hydrogen atom

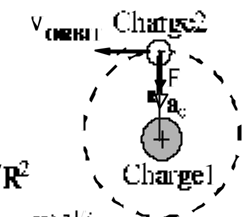
$$\Sigma F = ma = ma_c = mv^2/R$$

$$F = F_{Electrostatic} \sim \frac{\text{Charge 1 x Charge 2}}{R^2}$$

$v = v_{ORBIT}$

So, $m_{Charge2} v_{ORBIT}^2/R \sim$ Charge1 x Charge2 $/R^2$

Hence, $v_{ORBIT} \sim$ [Charge1 x Charge2 $/(m_{Charge2}R)]^{1/2}$



Here v, $v_{tangential}$, and v_{ORBIT} are the same thing.

9.2 Problems

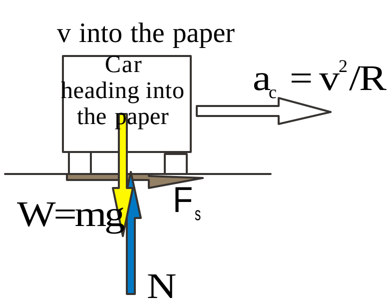
9.2.1`The Unbanked Highway

A car of mass m travels with a velocity v around a curve on a flat highway. The curve has of radius R. Show that for the car not to slip off the road the coefficient of static friction μ_s between the car tires and the road must satisfy the condition,

$$\mu_s >= v^2/(gR) ,$$

where g is the gravitational intensity (also called, the acceleration due to gravity).

Sketch the Free Body of Diagram of the car 20 points	State the Physical Principle(s) 20 points	Fill in the Details For each 30 points



Laws of Friction $F_s =< F_{smax} = \mu_s N > F_k = \mu_k N$ (1)

Here we are going to ignore he fact that the two tires travel in circle of different radii. In most practical situations R is in the range of 1000m while the tires are about 2 m apart. This puts your error under 1%. But if you are concerned with differential tire wear, you cannot neglect this difference.

Newton's 2nd Law

$+\wedge\Sigma F_{vertical} = 0$ $N = mg$ (2)

$+>\Sigma F_{horizontal} = ma = ma_c = mv^2/R$ $F_s = mv^2/R$ (3)

THE MATHEMATICS

Combing Eq.(1), (3) then (2) gives

$$mv^2/R = F_s =< F_{smax} = \mu_s N = \mu_s mg \quad (4)$$

Canceling m gives $v^2/R <= \mu_s g$. Hence

$$\mu_s >= v^2/(gR) . \quad (5)$$

INTERPRETATION

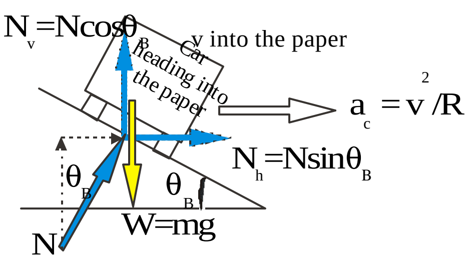
The result of Equation (5) is that the car travels in the circle of radius R at the speed v without sliding off the road so long as μ_s is equal to or greater than $v^2/(gR)$. For values of μ_s less than this value the car slides off the road seeking a circular path of larger R or must be slowed to a smaller velocity v.

9.2.2 The Banked Highway

A car of mass m travels with a velocity v around a curve on a highway. The curve has of radius R. The highway curve is banked an angle θ_B such that no friction is required to keep the car on the curve. The force that keeps the car on the curved path is the horizontal component of the normal force of the road. Show that the bank angle θ_B is

$$\tan \theta_B = v^2/(gR) .$$

Sketch the Free Body of Diagram of the car 20 points	State the Physical Principle(s) 20 points	Fill in the Details For each 30 points



$+>\Sigma F_{horizontal} = ma_c = mv^2/R;$ $\quad N\sin\theta_B = mv^2/R , \quad (1)$

$+\wedge\Sigma F_{vertical} = 0 ;$ $\quad N\cos\theta_B - mg = 0 , \quad (2)$

The components of N are as shown by the triangle then slid over to act from the road surface.

THE MATHEMATICS:

Eq. (2) becomes $\quad N\cos\theta_B = mg \quad (3)$

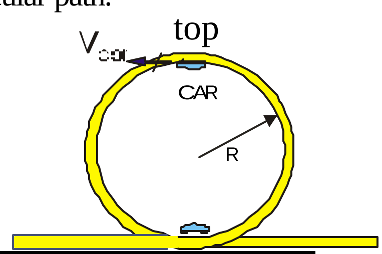
Dividing (1) by (3) gives $\quad N\sin\theta_B /N\cos\theta_B = (mv^2/R)/mg$

. Canceling N and m yields $\quad \tan\theta_B = v^2/(gR)$. $\quad (4)$

INTERPRETATION

The result of Equation (4) is that the car travel in the circle of radius R at the speed v without sliding off the road so long as $\tan\theta_B$ is equal to $v^2/(gR)$. For other values of $\tan\theta_B$ car slides up or down on the road seeking a different R or it velocity must change to stay on the initial circular path.

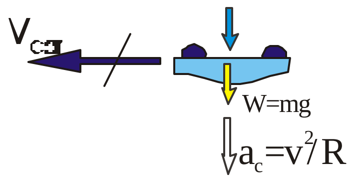
9.2.3 Don't fall off the top of the loop-the-loop



A car is traveling through a loop-the-loop as shown. If the car is not traveling fast enough it breaks contact the loop at the top. Show that the velocity of the car ,v , must satisfy the relation

$v^2 > gR$.

Sketch the Free Body of Diagram of the car 20 points	State the Physical Principle(s) 20 points	Fill in the Details For each 30 points



$+9\Sigma F_{horizontal} = ma_c = mv^2/R$;
the direction of the acceleration
is chosen as + for the convenience
of keeping the right side of the equation +.

$N + mg = mv^2/R$, (1)

The condition for staying in contact with the N>0
So at the top of the loop-the-loop $N = mv^2/R - mg > 0$ (2)

THE MATHEMATICS:

1. Eq. (2) after adding mg to both sides gives $mv^2/R > mg$
2. Now cancelling m and cross multiplying by R , we get the desired result

$$v^2 > gR . \quad (3)$$

INTERPRETATION

This problem applies to a class of situations. It applies to any situation that you are upside down at the top of a vertical circular path. Some examples are:

1 If you are flying upside down and Eq. (3) is true, then you do not need your seat belt to keep you in your seat. When $v^2 < gR$ you need your seat belt.

2 A motorcycle traveling a loop-the-loop.

Can you think of other situations?