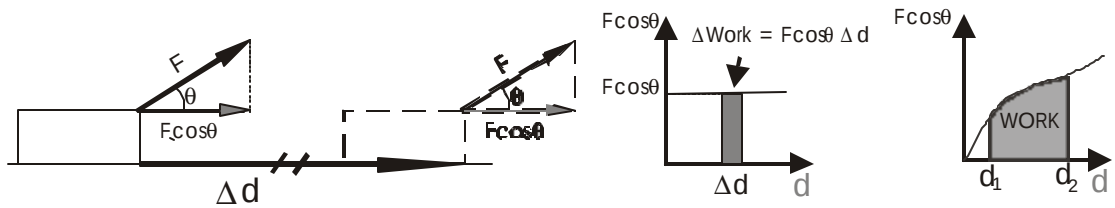
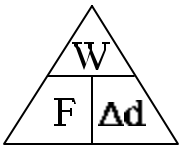
10. Mechanical Work, Power and Energy

10.1 Definition of Mechanical Work. Mechanical work **may** be done when a force acts on a body while that body undergoes a displacement. For a force to do work, it **must have a component parallel to the displacement** . Work is done only by the component of the force parallel to that displacement.

Work = **F**cosθ **Dd** = **F. Dd** (the scalar product) , [ML^2/T^2]
Viewing **F**cosθ **Dd** as an area element leads to Work = area under the **F**cosθ vs **Dd** curve.
Units: foot-pound (fp), Joule (=Newton meter), kilowatthour.



10.1.1 Units of Work and Energy - Definitions



1 **Joule (J)** = work done by a force of 1 N (Newton) applied for a displacement of 1 m (meter) parallel to the force (1 J = 1 Nm) .
1 **foot-pound (fp)** = work done by a force of 1 p (pound) applied for a displacement of 1 f (foot) parallel to the force .
1 kilowatt hour (Kwhr) = 1 kilowatt of power used for 1 hour = 3.6 x 10^6 J.

Let's convert from Joules to fp.

$$1J = 1Nm = 1N \times m = 1N\left(\frac{1}{9.81}\right)\frac{kg}{N}(2.206)\frac{p}{kg} \times m(100)\frac{cm}{m}\left(\frac{1}{2.54}\right)\frac{in}{cm}\left(\frac{1}{12}\right)\frac{f}{in} = 0.739fp$$

10.1.2 Examples:

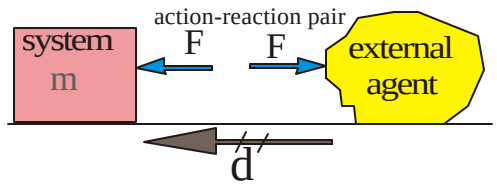
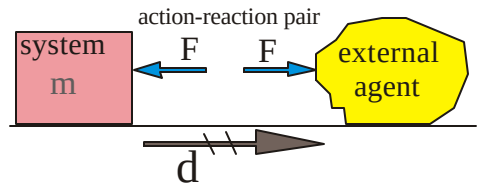
A 70 kg (154.4 p) person walks up 10 m (32.8 f), 2 to 3 stories, of stairs does 6867J of work. How many fp is this?

10.1.3 Work: Positive and Negative, Body and Environment, Done On and Against

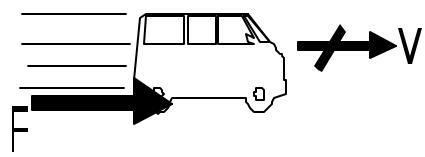
Work is done because of an interaction between two systems. One is the system under study, often a single body. The second system is the rest of the universe, often called the environment, and often consists of one external agent. This is because of Newton's 3rd Law of action and reaction. If an external agent applies a force of the body under study, then this applies an equal opposite force on the external agent. This raises the question of who does what to who, just like in Newton's 3rd Law. The picture below explains ths situation by illustration.

The picture shows the action-reaction pair acting between the system and the external agent and their displacement. Both simultaneously undergo the same displacement - same distance, same direction.

The key is the relationship between the directions of the force and of the displacement. This determines the nature of the work : Is it done "on" or "by"? Positive or negative? Against?

	
positive work is done **on** the system (m) by the environment **negative** work is done **on** the environment or the system does work **against** the environment **negative** work is done **by** the system. **positive** work is done **by** the environment	**negative** work is done **on** the system (m) by the environment or the system does work **against** the environment **positive** work is done **on** the environment by the system **positive** work is done **by** the system. **negative** work is done **by** the environment

10.2 Power - Definition

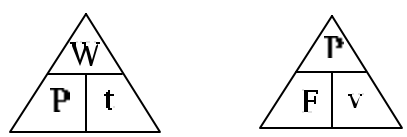


Power is the rate of doing work = work/time

$= \mathbf{F}\cos\theta\ \mathbf{Dd}/\Delta t = \mathbf{F}\cos\theta\ \mathbf{v} = \mathbf{F}.\ \mathbf{v}$

(the scalar product). $[ML^2/T^3]$

10.2.1 Units of Power - Definitions



1 **watt** = 1 Joule/sec (1 J/s)

1 **horsepower (1 hp)** = 33,000 fp/min = 550 fp/s

Now let's convert from hp to watts.

$$1\text{hp} = 550\frac{\text{fp}}{\text{s}} * \text{f}\,(12)\frac{\text{in}}{\text{sf}}(2.54)\frac{\text{cm}}{\text{in}}\left(\frac{1}{100}\right)\frac{\text{m}}{\text{cm}}\left(\frac{1}{2.206}\right)\frac{\text{kg}}{\text{p}}(9.8)\frac{\text{N}}{\text{p}} = 550*1.3547\frac{\text{Nm}}{\text{s}} = 745.4\text{watts}$$

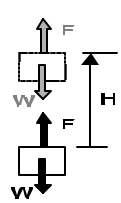
10. 2.2 Exercise:

1. Show that if a 70 kg person runs up 10 m of stairs in 10 s their mechanical power output is 6867J/10s = 686.7 watts. What is the hp?

10.3 Energy

10.3.1 Definition Energy is the ability to do work.
$[ML^2/T^2]$

Units: the same as work.



H=	1.0 m	10.0 m	100.0 m
m	PE (J)	PE (J)	PE (J)
1 kg	9.81	98.1	981
10 kg	98.1	981	9810
100 kg	981	9810	9.81E+04
1000 kg	9810	9.81E+04	9.81E+05

10.3.2.1 Potential Energy - the ability to do work because

of position.

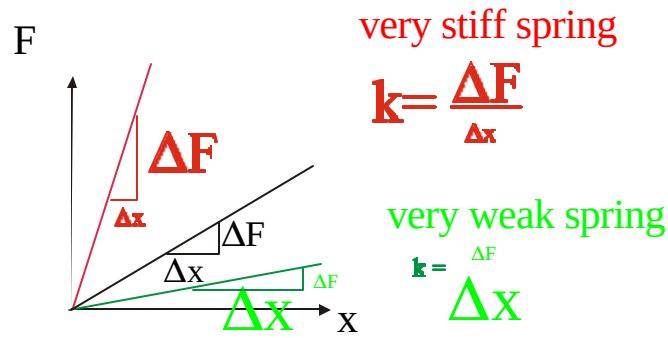
Local gravity P.E= mgH = WH = work done by the external force F to raise the body to the height H,

General gravitational P.E. = - GMm/r (see "Gravity" Section 12.3)

10.3.2.1.1 Exercise: By direct calculation, including using units, verify all of the values for PE in the above table.

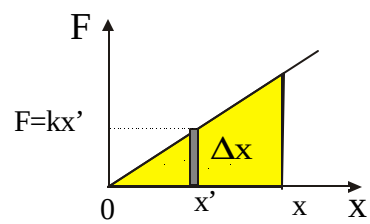
Spring P.E. = ½kx^2 - the work done by the external force F required to stretch or compress the spring by Δx.

According to the empirical Hooke's Law the relationship between an externally applied F on a spring and the total stretch or compression x of the spring is a linear relation given as F = kx , where k is called the spring stiffness constant, or simply, the spring constant.



The stiffer the spring the larger the value of k and the steeper the slope of the F vs. x line.

The work done to stretch or compress the spring by Δx , when the deformation is at x', is the rectangular area kx' high and Δx wide. kx' Δx. is an area element under the F vs x curve, hence a work element ΔWork. So, the total work done to deform the spring from 0 to x is the area under the F vs x curve from 0 to x. The curve is a triangle. Hence Work$_{0\text{->}x}$ = ½ height * base = ½ kx *x = ½kx^2



10.3.2.1.2 Exercise: 1. Calculate x = F/k to verify all values in the table on the left.

2. Calculate the energy to verify all values in the table on the right.

x = F/k

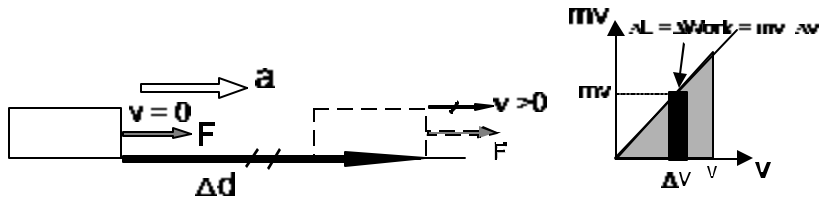
F=	1 N	10 N	100 N	1000 N
k (N/m)	x	x	x	x
10	0.1 m	1.0 m	10.0 m	100.0 m
100	0.01 m	0.1 m	1.0 m	10.0 m
1000	0.001 m	0.01 m	0.1 m	1.0 m
10000	0.0001 m	0.001 m	0.01 m	0.1 m

PE = Work = ½kx^2

x=	0.0001 m	0.001 m	0.01 m	0.1 m
k (N/m)	Energy (J)	Energy (J)	Energy (J)	Energy (J)
10	5.0E-08	5.0E-06	0.0005	0.05
100	5.0E-07	5.0E-05	0.005	0.5
1000	5.0E-06	0.0005	0.05	5
10000	5.0E-05	0.005	0.5	50

10.3.2.1.3 Exercise: Verify by direct calculation, including units, all of the values in the above two tables.

10.3.3.1 Kinetic Energy - the ability to do work because of motion. K.E. = ½mv^2.



Consider the work done to accelerate the mass m from v=0 to v > 0. To achieve this a force F, parallel to the displacement, is applied to the mass m over a displacement Δd. Hence F does work. You get it back when you try to stop the body.

$\Delta KE = KE_{final} - KE_{initial}$

ΔKE = ΔWork to accelerate the body from rest

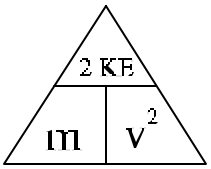
= F Δd, *force applied over the displacement during acceleration,*

= ma Δd, *because* **F** = m**a** - Newton's 2nd Law of motion,

= m (Δv/Δt)Δd, *since* **a** = Δ**v**/ Δt - by definition

= m Δv(Δd/Δt), *shifting* Δt to Δd

= m Δv v = mv Δv, *using* v = Δd/Δt - by definition.



mv Δv is the area under the mv vs v curve. So the work done to accelerate the body from v = 0 to v>0 is this triangular area from v = 0 to v. The area of a triangle is ½ base x height = ½ v x mv. Hence KE = ½ mv^2 .

Alternatively, you can **use calculus**. All of the Δ's become d's , so ,

dKE = F dx = m a dx = m dv/dt dx =

= m [dv(x)/dt] dx , v=v(x) *is a function of* x

= m [dv/dx] [dx/dt] dx *using the chain rule* ; dv(x)/dt=[dv/dx][dx/dt]

= m v dv/dx dx *using* v = dx/dt, by definition

= mvdv, *using the chain rule* ; dv(x) = dv(x)/dx dx ,

integrating.

$$=KE=\int_0^v mvdv=\frac{1}{2}mv^2$$

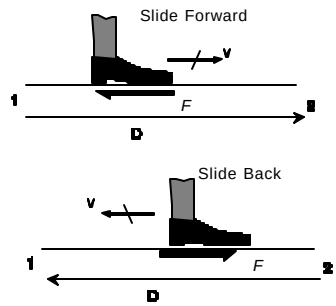
		adult humar	20 ton truck
	a bullet	at 3 mph	at 30 mph
mass (kg)	1.000E-03	7.000E+01	1.814E+04
velocity (m/s)	5.000E+02	1.341E+00	1.341E+01
KE (J)	1.250E+02	6.294E+01	1.631E+06
Momentum =mv (kg m/s)	5.000E-01	9.387E+01	2.433E+05

10.3.3.1.1 Exercise: By direct calculation, including units, verify the values of KE and Momentum in the above table.

10.3.4.1 Conservative Forces are forces whose value and direction depends only on position. When you do work against a conservative force by traveling around a closed path, (you end at your starting

point), the net work is zero: That is, $W_{A->A} = 0$, for the work done going from position A that returns to position A, regardless of path. So, the work done from displacements A to B, $Work_{A->B}$ depends only on the end points, not on the path. Because of this, work done against a conservative force can be expressed as a <u>change in Potential Energy</u>. Further, the value of this work function at any point in space can be taken as the Potential Energy at that point, PE(x,y,z).

10.3.4.2 Non-Conservative Forces are forces that are not conservative. That is they are path dependent. An example are dissipative forces. When you do work against a dissipative force by traveling in a closed path, the work is not zero - $WORK_{A->A}$ =/ 0, These <u>cannot</u> be expressed in as a potential energy. For example, suppose you slide your foot forward on the floor by two feet, then slide it back two feet, returning to the original position, hence forming a closed path. In this closed path you have done twice the work as you did when slid it forward. This is because the friction force always acts opposite to the direction of motion.



$$Work_{1->.1} = Work_{1->2} + Work_{2->.1} = - FD + -FD = -2FD = 2W_{1->2}.$$

The "-" sign is because F and D are in opposite directions, so the work is done by you against friction, not on you by friction.

10.3.5.1 Work-Energy Theorem - The change in kinetic energy from A (initial) to B (final) is the work done on the system by the environment (all of the external forces);

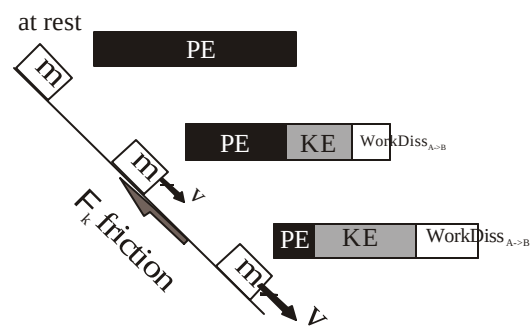
$\Delta KE = KE_A - KE_B$ = Work done on the system as it goes from A->B

= Work done by conservative external forces + Work done by external non conservative forces

$= Work_{conservative\ A->B} + Work_{non\text{-}conservative\ A->.B}$

$= PE_A - PE_B + Work_{non\ conservative\ A->B}$

$= PE_A - PE_B + Work_{nc\ A->B}$



where the PE's are the potential energies arising from the conservative external forces. The final result of this is

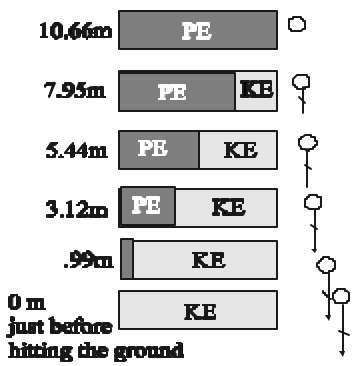
$$\overset{\text{Initial}}{K.E._A + P.E._A} = \overset{\text{Final}}{K.E._B + P.E._B} + Work\ Dissipative_{A->B}.$$

Keep in mind that this is the work is done against the body. Hence it is negative, so the body loses energy.

10.3.5.1.1 Conservation of Energy - a system moves only against conservative forces, then its total energy remains constant. That is

$$K.E._A + P.E._A = K.E._B + P.E._B = \text{const.}$$

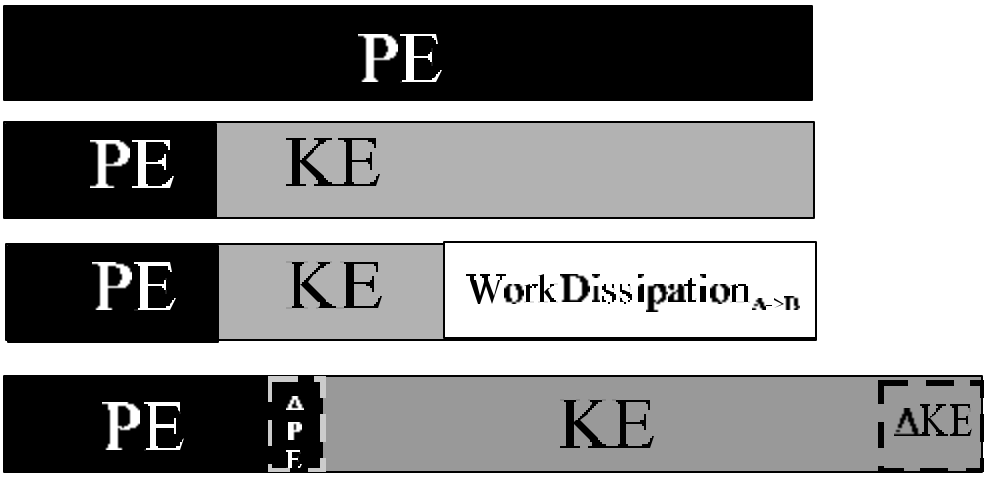
10.3.5.1.1 Example2: Free Fall - A ball is dropped from 10.6555 m (precision only for calculations) above the ground.



height	PE	KE	velocity	PE/E	KE/E
10.66 m	104.42 J	0.0 J	0.0 m/s	1.000	0.000
9.28 m	90.94 J	13.48 J	5.19 m/s	0.871	0.129
7.95 m	77.93 J	26.5 J	7.28 m/s	0.746	0.254
6.67 m	65.38 J	39.04 J	8.84 m/s	0.626	0.374
5.44 m	53.31 J	51.12 J	10.11 m/s	0.511	0.489
4.26 m	41.71 J	62.72 J	11.2 m/s	0.399	0.601
3.12 m	30.57 J	73.85 J	12.15 m/s	0.293	0.707
2.03 m	19.91 J	84.51 J	13.0 m/s	0.191	0.809
0.99 m	9.72 J	94.7 J	13.76 m/s	0.093	0.907
0.0 m	0.0 J	104.42 J	14.45 m/s	0.000	1.000

10.3.5.2 The Work Energy Box

This is a graphic, used in the above examples, that shows the PE, KE and Work done on the physical system as it progresses through its stages. If the work done is against the system (negative work done on the system), either friction or any force acting to slow the system, the box stays the same size. This is also called work done by the system. If positive work is done on the system, then the box can become larger because system has an increase in its ability to do work, its energy. The figure below illustrates these situations.



It starts with PE only.

No non-conservative forces

A dissipative force (non-conservative) acting.

Work other added to PE and KE . Aiding force acting.

An example of the last box might be a initially moving up a frictionless inclined plane while being above the bottom that suddenly has an external force pushing it up plane.

10.3.5.2.1 Spring PE Lost to Friction

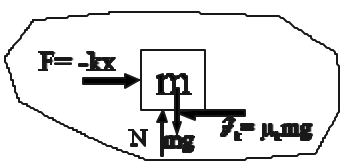
A body of mass m = 0.5 kg is held against a spring compressed by x_1 = 0.05 m. The spring stiffness is k = 1000 N/m. It is released from rest on a rough horizontal plane with coefficient of static friction μ_s = 0.6 and kinetic friction μ_k = 0.5. How far does it travel before coming to rest.

This problem has three separate situations.

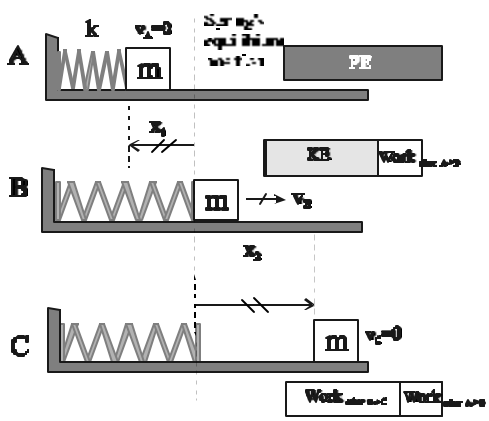
1. The first is just as the block is released while against the compressed spring.
2. The second is when it breaks away from the now uncompressed spring.
3. The third, and final, situation is when the block comes to rest on the rough horizontal plane.

But, before we do anything we must find out whether the block moves: Is $F_{spring} > F_{s\,MAX}$?

To assess the influence of friction, sketch a Free Body Diagram of the Block when in contact with the spring.



Sketch the block in each of its three situations..



1. We get N from Newton's Laws of Motion:

Principles:	**Details**	**Result**	
$+\uparrow\Sigma F_{vertical} = 0$; N-mg = 0.		So, N=mg.	(1)

2. Analysis of the spring force gives:

Principles:	**Details**	**Result**
$F_{spring} = kx$	$F_{spring} = 10^3$ N/m *5x10^{-2}m =50N	

Principles: The Laws of Friction

$F_s =< F_{s\,MAX} = \mu_s N > F_k = \mu_k N = \mu_k mg$. (2)

Details

$F_{s\,MAX} = \mu_s N = \mu_s mg = 0.6*0.5kg*9.8m/s^2 = 2.9N$

Result So, $F_{spring} = 50N > 2.9N = F_{s\,MAX}$, **it moves**.

From A-> B

State Physical Principles

$KE_A + PE_A = KE_B + PE_B + Work_{Other\ A->B}$

Fill in the Details

$0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_B^2 + 0 + \mu_k N x_1$ (3)

at rest spring compressed spring free energy lost to friction

From B->C

State Physical Principles

$KE_B + PE_B = KE_C + PE_C + Work_{Other\ B->C}$

Fill in the Details

$\frac{1}{2}mv_B^2 + 0 = 0 + 0 + \mu_k N x_2$ (4)

Combining Eq(3) and (4) eliminates $\frac{1}{2}mv_B^2$ and replacing N with mg from Eq(1) gives

$$\tfrac{1}{2}kx_1^2 = \mu_k mgx_2 + \mu_k mgx_1 \qquad (5).$$

Solving (5) for x_2 yields

$$x_2 = \frac{kx_1^2}{2\mu_k mg} - x_1 = \frac{1x10^3 \frac{N}{m}(5x10^{-2}\,m)^2}{2x0.5x0.5kgx9.8\frac{m}{s^2}} - 5x10^{-2}m = 0.510 - 0.050m = 0.460m$$

Notice, as indicated by the energy boxes in the figure, that at point A the energy is all PE. When it gets to B, it has lost energy to friction. The remaining energy is all KE. When it gets to C it has lost its remaining energy to work against friction.

10.3.5.3 Force and Energy Gradient F = -DE/Dx

The mechanical energy of a system is increased by doing work on it, and decreased by doing work against it. So,

$$-\Delta E = \text{Work}_{against} = F_{against}\,\Delta x\,.$$

Hence,

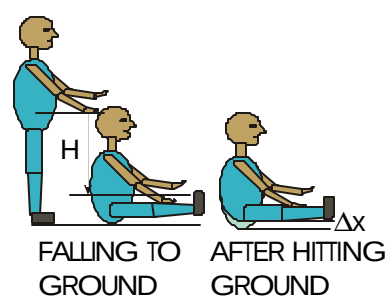
$$F_{against} = F = -\Delta E/\Delta x \quad .$$

This is the force that decreases the energy, hence $-\Delta E$, as the body moves a displacement Δx.

10.3.5.3.1 Examples:

In both of the examples below, ΔE is the potential energy resulting from the body's center of mass falling a height H **before** hitting the ground. This causes the body to have a velocity just before hitting the ground because the conversion of PE to KE through its fall by the height H. The body is stopped by traveling a distance Δx **after** hitting the ground.

10.3.5.3.1a The Butt Flop One may say "Well, so what?" Okay. Let's look at a very common situation. A very small child literally drops on its rump (behind, posterior) with all its weight, immediately gets up and charges off to whatever he or she was doing before. A fully grown adult undergoes the same fall and winds up in the hospital with a severe back injury, perhaps even paralyzed from the waist down for the rest of their life. In this situation the **D**x, the distance "fallen" **after** hitting the ground, is the thickness of the fatty padding of the buttocks, about 1 cm = $1x10^{-2}$ m. The basic situation is that the person's whole weight falls a distance H, of about half their height, **before** hitting the ground. The energy to be degraded is the potential energy PE = mgH, where H is about half the height of the person. So,



$$F = -\,mgH/\Delta x \quad .$$

For the small child; m=10 kg (22 p) and is about 0.6m . (24 in) tall. So the fall distance is H = 0.3 m,

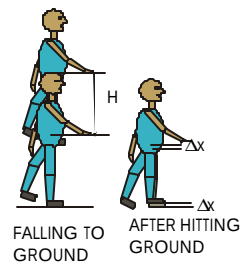
$$F = -\,10kg \times 9.8m/s^2 \times 0.3\,m\,/1x10^{-2}m = -2.94x10^3\,N \text{ (about 660 p).}$$

For the big tall adult; m = 100 kg (220p) and about tall adult 2m (6 f, 6 in) tall.. So, H=1m,

$F = -100kg \times 9.8m/s^2 \times 1.0\ m / 1\times10^{-2}m = -9.8\times10^4\ N$ (about 22,000 p) .

The linear scale factor is 0.3, that is the child's length is 0.3 that of the big tall adult. We will see later when dealing with stress that the stress on the supporting spinal column is about $9\times10^5\ N/m^2$ for the small child, but about 2×10^7 for the big tall adult - over 20 times larger!

. **10.3.5.3.1b The Step Off the Curb Mistake** Most of us have had the situation where when stepping off a curb, going from sidewalk to street, you got a back breaking jolt. You misjudged the height of the curb distance. You stiffened the forward leg and shifted your weight from the backward leg to the forward leg. But, the forward leg had not reached the street yet. OOPS! You went into a short free fall - your weight fell about H = 8 i (0.2 m) as you stiffened leg made contact with the street. OUCH! The Δx is the stopping distance after you hit the ground(street). That's about 2 mm (2×10^{-3} m) between the shoe heel material compressing and some foot bending. For a 70 kg person we get



$F = -70kg \times 9.8m/s^2 \times 2\times10^{-1}\ m / 2\times10^{-3}m = 6.9\times10^4\ N$ (about 15,500 p) .

10.4 Rotational Kinetic Energy of a Rigid Body and Moment of Inertia (again)

10.4.1 Rotational K.E. = ½Iw² - Derivation

$K.E. = ½\Sigma(\Delta m_i v_i^2)$; summed over all mass elements Δm_i

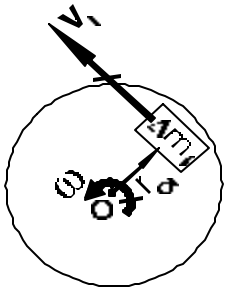
$= ½\Sigma\Delta\ m_i(r_o\omega)^2$; linear velocity $v_i = r_{oi}\omega$

$= ½\Sigma\Delta\ m_{oi}r_i^2\omega^2$; carry the square through

$= ½(\Sigma\Delta m_i r_{oi}^2)\omega^2$; for a rigid body every has element has the same angular velocity ω

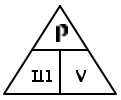
$= ½\ I_o\omega^2$; isolating and defining I_o.

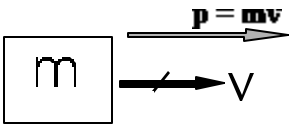
where I is the **moment of inertia about o.** $\mathbf{I_o = \Sigma\Delta m_i r_{oi}^2}$.



11.1 Linear Momentum p

Definition **p** = m**v** [ML/T].





			adult human	adult human	20 ton truck
	a fly	a bullet	at 3 mph	falling 10 m	at 30 mph
mass (kg)	2.000E-05	1.000E-03	7.000E+01	7.000E+01	1.814E+04
velocity (m/s)	2.000E-01	5.000E+02	1.341E+00	1.400E+01	1.341E+01
KE (J)	4.000E-07	1.250E+02	6.294E+01	6.860E+03	1.631E+06
Momentum (kg m/s)	4.000E-06	5.000E-01	9.387E+01	9.800E+02	2.433E+05

11.1.1 Exercise: By direct calculation verify the energy and momentum values in the above table.

11.2 Newton's 2nd Law Revisited

$\mathbf{F} = m\mathbf{a} = m\,\Delta\mathbf{v}/\Delta t = \Delta(m\mathbf{v})/\Delta t = \Delta\mathbf{p}/\Delta t = d\mathbf{p}/dt.$

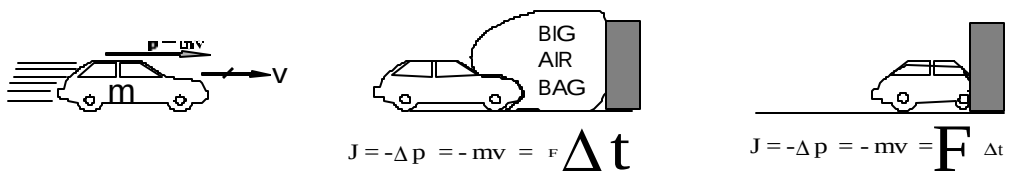
Force = the rate of change of momentum.

11.3 Linear Impulse $\mathbf{J} = \mathbf{F}\,\Delta t = \int \mathbf{F}dt$ [ML/T]

11.3.1 Impulse-Momentum Theorem $\mathbf{J} = \Delta\mathbf{p}$, the net external impulse acting on a system equals the change in momentum of that system. But $\mathbf{J} = \mathbf{F}t$. For situations where one has the same $\Delta\mathbf{p}$, then

$\Delta\mathbf{p} = \mathbf{J} = \mathbf{F}\Delta t.$,

or $\Delta\mathbf{p} = \mathbf{F}\,\Delta t = \mathbf{F}\,\Delta t$ for variations in the sizes of $\mathbf{F}$ and Δt. The car crashing into the wall in the picture below and example illustrate the meaning of this.



11.3.1.1 Example:

A 70kg person falls 10 m (about 33 f). From the table above, the momentum = 980 kg m. If the person stops suddenly at the bottom in a time $t = 10^{-2}$ sec, the stopping force is

$$F = \Delta p/\Delta t = 980\ (\text{kg m/s})/10^{-2}\text{s} = 9.8\text{x}10^{2}/10^{-2}\ (\text{kg m/s}^2) = 9.8\text{x}10^{4}\ \text{N (about 22,000 p)}$$

If they stop over the much longer time t = 1 sec because they fell on a thick air mattress, then

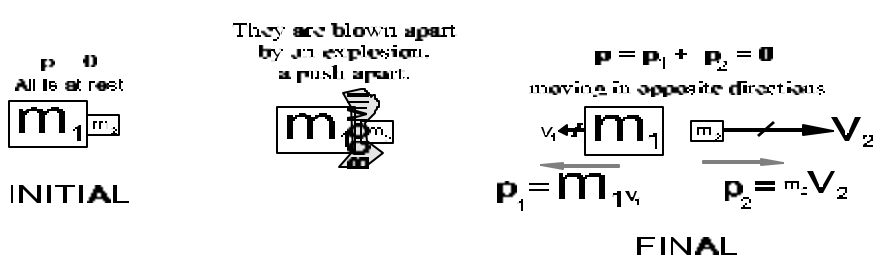
$$F = \Delta p/\Delta t = 980\ (\text{kg m/s})/1\text{s} = 9.80 \text{ x } 10^{2}\ (\text{kg m/s}^2) = 9.8\text{x}10^{2}\ \text{N (about 220 p)}$$

If they stop over a time t = 5 sec because as the hit the ground they collapsed and rolled (the paratrooper's fall) , then

$$F = \Delta p/\Delta t = 980\ (\text{kg m/s})/5\text{s} = 49\ \text{N (about 44 p)}$$

11.4.1 Conservation of Linear Momentum - when a system is subjected to no net external <u>impulse</u>, then the linear momentum of that system remains constant. These are collisions and recoils.

$$\mathbf{p}_{\text{initial}} = \mathbf{p}_{\text{final}}$$



11.4.1.1 Example:

Preview, Review Summary - University Physics I

Dr. Hulan E. Jack Jr.

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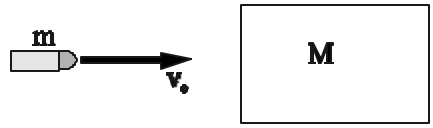
A bullet of mass m = 0.01 kg with a velocity v_o = 500 m/s hits a stationary block of mass M = 0.99 kg. The bullet embeds in the block and the combined bullet and block move of with velocity v. What is the velocity v? How much energy is lost? 30 points



Sketch the situation just after this collision

State the physical principle(s)

Conservation of linear momentum

$p_{initial} = p_{final}$

Fill in the details.

$$mv_o + M(0) = (M + m)v \qquad (1)$$

At rest Moving as one

The KE Lost = ΔKE = KE_{Final} - $KE_{Initial}$

The initial KE , $KE_{initial}$ is

$$KE_{initial} = ½mv_o^2 = ½*1x10^{-2}kg*(5x10^2m/s)^2 = 0.5*10^{-2}*25x10^4kgm^2/s^2 = 12.5x10^2\ J = 1.25x10^3\ J \qquad (2)$$

The value of the velocity just after collision is from Eq.(1)

$$v = mv_o/(M+m) = 1x10^{-2}kg * 5x10^2m/s/(0.01+.99)kg = (5\ m/s)/1 = 5\ m/s. \qquad (3)$$

The final KE, KE_{final} in terms of the intiial KE comes first, from the definition of KE in terms of v , M and m, then replacing v with Eq.(3). Finally do some algebra as follows

$$KE_{final} = \frac{1}{2}(M+m)v^2 = \frac{1}{2}(M+m)(\frac{mv_0}{M+m})^2 = \frac{1}{2}(M+m)\frac{m^2v_0^{\ 2}}{(M+m)^2} = \frac{m}{(M+m)}\frac{1}{2}mv_0^{\ 2} = \frac{m}{(M+m)}KE_{initial} \qquad (4)$$

$$KE_{final} = (m/(M+m)KE_{initial} = 10^{-2}/(0.01+0.99)\ KE_{initial} = 1x\ 10^{-2}KE_{initial} = 1x\ 10^{-2}*1.25x10^3 = 12.5\ J$$

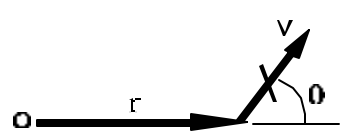
The change in KE , $\Delta KE = KE_{Final} - KE_{initial}$ in terms of $KE_{initial}$ is as follows as

$$\Delta KE = KE_{Final} - KEe_{initial} = [m/(M+m) - 1]KE_{initial} = ([m - M - m)/(M+m)]KE_{initial}$$
$$= [M\backslash(M+m)]KE_{initial}.= [\ 1\ /(1+\{m/M\})]KE_{initial}$$

11.5 Angular Momentum $L_o = I_o\ \omega$ for a rigid body

[ML^2/T]

$= \mathbf{r_o} \times \mathbf{p} = \mathbf{r_o} \times m\mathbf{v} = m\,\mathbf{r_o} \times \mathbf{v} = mvr\sin\theta$ for a particle.



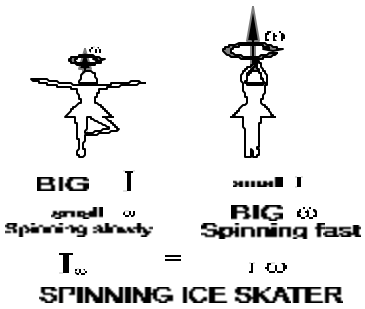
11.6 Angular Impulse $\mathbf{J} = \mathbf{t}\ \Delta t = \mathbf{r} \times \mathbf{F}\ \Delta t$

11.7 Angular Impulse-Momentum Theorem

$\mathbf{J} = \Delta\mathbf{L}$ --> $\mathbf{t}\ \Delta t = \Delta(I\ \omega)$ for rigid bodies

$\mathbf{r} \times \mathbf{F}\ \Delta t = \Delta(m\ \mathbf{r_o} \times \mathbf{v}) = m\ \Delta\ \mathbf{r_o} \times \mathbf{v} = m\ d\mathbf{A}/dt$ for a particle

11.8 Conservation of Angular Momentum



When a system is subjected to no net external angular impulse , then its angular momentum is conserved:

If $\mathbf{J} = \mathbf{0}$, then $\Delta\mathbf{L} = \mathbf{0}$.

$$I\omega = \text{constant}, \text{ or } I_1\ \omega_1 = I_2\ \mathbf{w}_2$$