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14. Simple Harmonic Motion

14.2 SHM - The Mathematical Model

Simple Harmonic Motion is describes by the equation

$$x = A \sin 2\pi ft = A \sin 2\pi t/T,$$

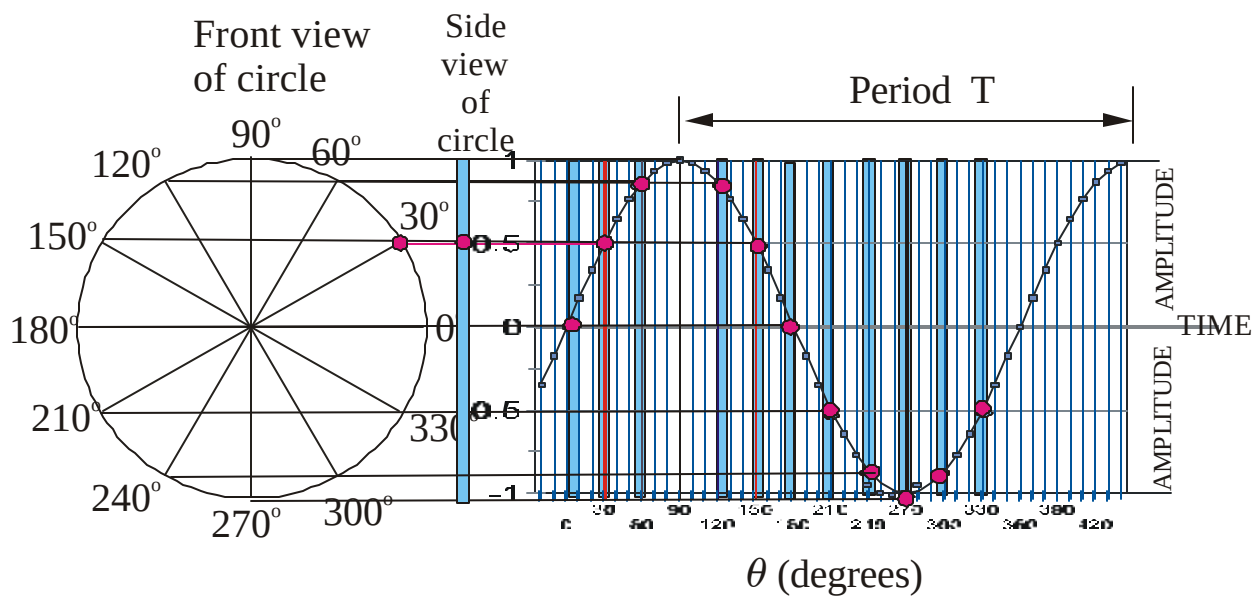
where

x is the displacement or position of the body with time. It is the dependent variable.
t is time. It is the independent variable.
A in the amplitude, the maximum and minimum values of x.

T [T] is the **period,** the time that it takes to complete one cycle.
f [1/T] is the **frequency**, the cycles per unit time.

The period and frequency are inversely related -

$$f = 1/T, \quad T = 1/f \ .$$



A marker is put on the rim of a circle rotating with a constant angular velocity ω. Turning this circle to its side view and pull it to the right at a constant linear velocity, the marker sweeps out the sin curve as shown.

$$v = dx/dt = 2\pi f A\cos 2\pi ft$$

$$a = dv/dt = d^2x/dt^2 = 2\pi f(-2\pi f A\sin 2\pi ft) = -(2\pi f)^2 A\sin 2\pi ft = -(2\pi f)^2 x.$$

So

$$a = -(2\pi f)^2 x$$

The relations acceleration = $-(2\pi f)^2$ displacement

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is the **characteristic equation for simple harmonic motion (SHM)**

14.2 SHM Physical System - Physical Models

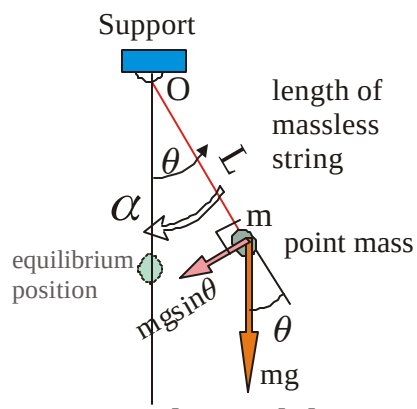
Physically, simple harmonic motion arises from physical systems that have linear forces forcing a displacement back to its equilibrium position whenever it is displaced from that position. The equilibrium position is a position where the system will stay if left alone undisturbed. The linear force pulling it back to its equilibrium position is called a linear **restoring force.** The restoring force always acts in the direction opposite to the displacement.

14.3 The Simple Pendulum

A point mass m hanging from a massless string of length l swings back and forth over a small angle 2_{Max}.

Newton 2nd Law for Rotation $\tau_O = I_O \alpha$.

The torque is $\tau_O = - Lmg \sin\theta$
and $I_O = mL^2$.



a simple penduluum

"-Lmg sinθ " because this is a **restoring torque** that acts opposite to θ; it attempts to return the pendulum to its equilibrium position. $\tau_O = I_O \alpha$ yields

$$- Lmg \sin\theta = mL^2\alpha \quad .$$

Using the the **small angle approximation** (Sec. 4.4), mg sinθ becomes mg θ . After canceling m and an L gives

$$\alpha. = -(g/L)\theta$$

angular acceleration = - (g/L) displacement.

Comparing this with the mathematical model gives

acceleration = $- (2\pi f)^2$ displacement

the coefficients of the displacement must be the same, so

$$(2\pi f)^2 = g/L .$$

Hence

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Simple Pendulum frequency, f and Period , T, for small angle approximation vs L

L=	0.1 m	0.2 m	0.3 m	0.5 m	1.0 m	2.0 m	3.0 m	10.0 m
T (sec)	0.634 s	0.897 s	1.099 s	1.419 s	2.006 s	2.837 s	3.475 s	6.344 s
f (Hz)	1.576	1.115	0.910	0.705	0.498	0.352	0.288	0.158

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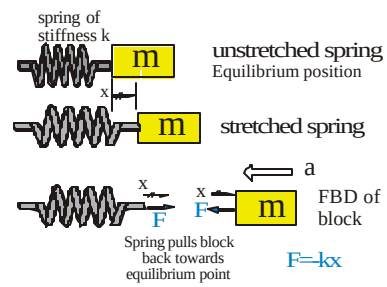
14.4 The Spring Pendulum

A body of mass m is attached to a spring with stiffness constant k. The body slides along a frictionless surface as swings back and forth over a maximum displacement x_{Max}.

Newton's 2nd Law $F = ma;$
Hooke's Law for the spring is $F = kx$
these give the result $-kx = ma.$



The Spring Pendulum

The force F acts in the opposite direction to the displacement. It always pulls the mass back to the equilibrium (unstretched) position. So, this force is called a **restoring force**. This gives

$$a = -(k/m)\,x.$$

Again, comparing this with

$$\text{acceleration} = -(2\pi f)^2 \text{ displacement}$$

the coefficients of the displacement must be the same, so

$$(2\pi f)^2 = k/m\,.$$

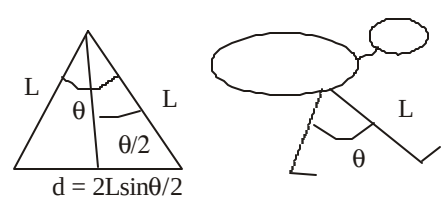
Hence

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Spring Pendulum frequency f and period T for values of m and k

k (N/m)	m(kg)=	0.100	0.200	0.300	0.500	1.000	2.000	3.000	10.000
1	T (sec)	1.99	2.81	3.44	4.44	6.28	8.89	10.88	19.87
	f (Hz)	0.503	0.356	0.291	0.225	0.159	0.113	0.092	0.050
10	T (sec)	1.41	1.19	1.07	0.94	0.79	0.67	0.60	0.45
	f (Hz)	0.709	0.844	0.934	1.061	1.262	1.500	1.660	2.243
100	T (sec)	0.529	0.577	0.607	0.647	0.706	0.770	0.810	0.941
	f (Hz)	1.890	1.733	1.647	1.545	1.417	1.299	1.235	1.063
1000	T (sec)	0.273	0.262	0.255	0.247	0.237	0.226	0.221	0.205
	f (Hz)	3.661	3.823	3.922	4.049	4.228	4.415	4.529	4.882

14.5 Size Effect Little Legs Swing Faster But Carry The Body Slower.



The speed with which a creature would "naturally" travels is determined by the frequency , f (= 1/T), with which its legs would "naturally" oscillate times the length, d, of its pace. Let's say that L is the length of the legs and the legs spread about 30° in taking a step. The triangle of the pace in the picture is an isosceles triangle with the side opposite θ/2 having length Lsinθ/2 . So, the pace length, d, is 2Lsinθ/2 . The velocity, v, is

$$v = d/t = d/T = d\,f\,,$$

because the time t of a pace is the period of oscillation T, hence 1/f .

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If we use the frequency for the simple pendulum as our model, then the speed v will be

$$v = f\,2L\sin(\frac{\theta}{2}) \approx 2fL\sin(\frac{30^o}{2}) = 2(\frac{1}{2\pi}\sqrt{\frac{g}{L}})L\sin(15^o) = \frac{\sin 15^o}{\pi}\sqrt{Lg}$$

So, using this model $v \approx L^{1/2}$.

Hence, the creature with little legs will have legs a flying while not traveling very fast. Watch an ant or any other bug. By comparison watch a horse. This model applies only to walking. It does not apply if the creature breaks into a trot (or a jog), or a gallop (or a run).

Velocity of the creature vs "size" L of the creature

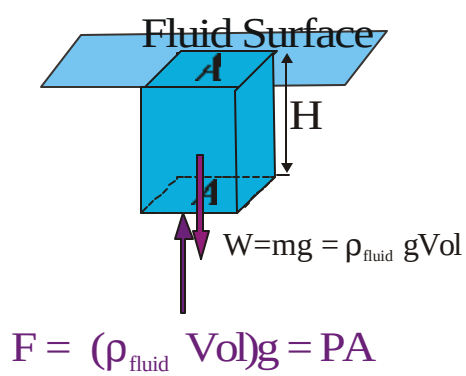
L=	0.001 m	0.01 m	0.1 m	0.2 m	0.5 m	1.0 m	1.5 m	5.0 m
v (m/s)	8.16E-03	2.58E-02	8.16E-02	1.15E-01	1.82E-01	2.58E-01	3.16E-01	5.77E-01
v(f/s)	2.68E-02	8.47E-02	2.68E-01	3.79E-01	5.99E-01	8.47E-01	1.04E+00	1.89E+00
v(mph)	1.83E-02	5.77E-02	1.83E-01	2.58E-01	4.08E-01	5.77E-01	7.07E-01	1.29E+00

These values are about an order of magnitude too small. Observe your own walk. Watch others walk. You notice that there is swinging about the knee and hip joints. So, there are two pendula. The upper leg pivots about the hip joint, and the lower leg pivots about the knee joint. Attached pendula like this are called a coupled pendulum.. The coupled pendulum uses shorter L for frequency, hence higher frequencies, while keeping a larger L for distance per pace. This will increase the above velocities.

15. Fluids

15.1 Fluid pressure - the fluid pressure P at a depth H below the surface of a fluid is

$$P = \rho_{fluid}\, g\, H,$$



where g is the acceleration due to gravity and ρ_{fluid} is the mass density of the fluid. The mass density is assumed to be constant throughout the fluid - almost true for most liquids, very untrue for gases.

This comes from the force needed to support the column of fluid H deep with cross section area V. This force is the weight W= mg of the fluid in the column. The mass m = ρ_{fluid} Vol. The volume Vol = HA . The pressure P is defined as P=F/A. So,

$$P = F/A = mg/A = (\rho_{fluid}\,Vol)\,g/A = \rho_{fluid}\,gHA/A = \rho_{fluid}\,gH\,.$$

Water Pressure vs Depth H (Fresh water)					
H=	1.0E+0 m	1.0E+01 m	1.0E+02 m	1.0E+03 m	1.0E+04 m
P (N/m^2)	9.81E+03	9.81E+04	9.81E+05	9.81E+06	9.81E+07
P (p/in^2)	1.42	14.22	142.25	1422.45	14224.50
P (atmos)	9.68E-02	9.68E-01	9.68E+00	9.68E+01	9.68E+02

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5.2 Pascal's Principle - Pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid and the walls of the container.
Example: the hydraulic press. MORE LATER.

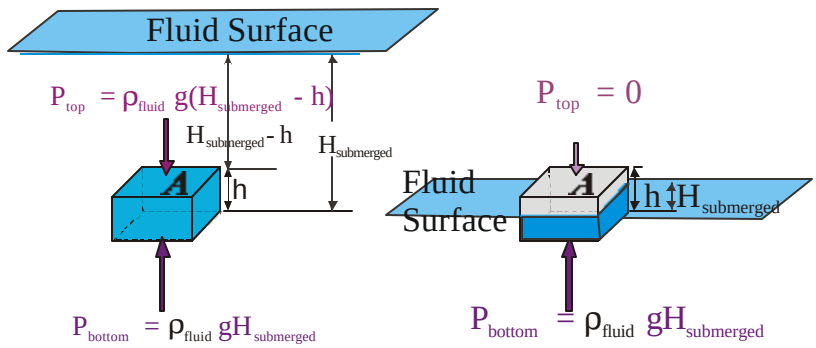
15.3 Buoyant force is the weight of the fluid displaced by a body in that fluid

Imagine a body with horizontal area A and height h. Its bottom is submerged a depth $H_{submerged}$ below the surface of the fluid. The pressure of this fluid pushing up on the bottom is $P_{bottom} = \rho_{luid} g H_{submerged}$. Hence causes an upward force

$F_{bottom} = PA = \rho_{luid} g H_{submerged} A$.

The buoyant force $F_{buoyant}$ is the net upward force given by

$F_{buoyant} = F_{bottom} - F_{top}$.



For the **floating body** top is out of the fluid, so the fluid pressure pushing down on the top is $P_{top} = 0$. Hence $F_{top} = 0$. So,

$F_{buoyant} = F_{bottom} - F_{top} = F_{bottom} = \rho_{luid} g H_{submerged} A = \rho_{luid} g Vol$.

For the **submerged body** there is fluid on top pushing down on the top with pressure $P_{top} = \rho_{fluid} g(H_{submerged} - h)$ producing $F_{top} = P_{top} A$. The buoyant force now becomes,

$\mathbf{F_{buoyant}} = F_{bottom} - F_{top} = \rho_{luid} g H_{submerged} A - \rho_{fluid} g(H_{submerged} - h)A$

$= \rho_{fluid} g A h = \mathbf{r_{fluid} g Vol_{submerged}}$.

$= \rho_{fluid} g h_{submerged} A =$

This is just the weight of the fluid displaced by the body.

15.4 Archimedes Principles

1. Submerged bodies displace an amount of fluid equal to their own volume (**two things cannot occupy the same space**)

   $(F_{buoyant} = \rho_{fluid} g Vol_{submerged} = \rho_{fluid} g A h)$

2. A floating body displaces an amount of fluid equal to its own weight (**the bouyant force must equal the weight of the body so that the net force is zero**). The body must be in equilibrium.

   $(F_{buoyant} = m_{body}\, g = \rho_{body} g Vol_{body} = \rho_{fluid} g Vol_{submerged} = \rho_{fluid} g\, A\, H_{submerged})$

Examples:

1. **Fraction of floating bodies of submerged.**

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For a floating body the submerged volume, $Vol_{submerged}$ is a fraction of volume of the body, Vol_{body} . First, by Archimedes 2nd Principle, a floating body displaces its own weight of fluid, so $F_{buoyant} = m_{body}\, g = \rho_{body} g Vol_{body}$. But, the buoyant force $F_{buoyant}$, is always $F_{buoyant} = \rho_{fluid} g Vol_{submerged}$. So combining the two yields,

$$\rho_{body} g Vol_{body} = \rho_{fluid} g Vol_{submerged} .$$

Hence

$$Vol_{submerged} = (\rho_{body}/\rho_{fluid})\, Vol_{body} .$$

(A) For an iceberg , the body is ice, the fluid, water . Their densities are related as $\rho_{ice} = 0.9\, \rho_{water}$. So

$$Vol_{submerged} = (\rho_{body}/\rho_{fluid})\, Vol_{body} = 0.9\, Vol_{body}$$

So, 90% of the iceberg is submerged!!!

(B) When $r_{body} = r_{fluid}$. When this is true, $(\rho_{body}/\rho_{fluid}) = 1$, so $Vol_{submerged} = Vol_{body}$. That is the surface of the body is just at the surface of the fluid.

2. **Submerged bodies - rise, fall or stay.**

There are two forces acting on the on the submerged body, the upward buoyant forrce, $F_{buoyant}$ and the downward weight W= mg. Newton's 2nd Law gives

$$up + \Sigma F = F_{net} = F_{buoyant} - mg = ma .$$

There are three cases:

(1) If $F_{net} = F_{buoyant} - mg > 0$, the body accelerates upward,
(2) if $F_{net} = F_{buoyant} - mg < 0$, the body accelerates downward,
(3) if $F_{net} = F_{buoyant} - mg = 0$, the body stays at present depth.

So, the controlling item is

$$F_{net} = F_{buoyant} - mg = \rho_{fluid} gAh - \rho_{body} gAh = (\rho_{fluid} - \rho_{body}) gAh .$$

This yields for the three cases

(1) $\rho_{fluid} > \rho_{body}$, the body accelerates upward,
(2) $\rho_{fluid} < \rho_{body}$, , the body accelerates downward,
(3) $\rho_{fluid} > \rho_{body}$, the body stays at present depth.

15.6 Surface Tension

TO BE ADDED AS AN ADDENDUM

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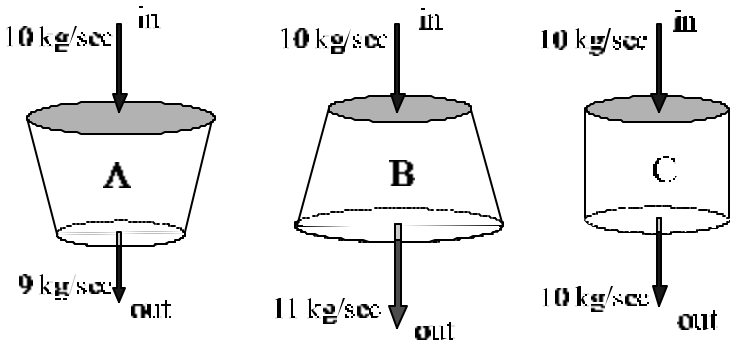
15.7 Fluids in Motion

15.7.1 Equation of Continuity - conservation of matter

Flow Machines - The argument of elimination

We have a tube open at both ends through which fluid can freely flow into it one end and out of it at the other. We have three possible consequences for the long term time rate of flow of matter through this tube. At least one must be true. We cannot a priori prove any one of them to be true. But, we can prove two to be false. Therefore, the remaining one must be true.



The sole effect of the tube is to influence the relation between the of the flow of matter into and out of them. Examples of three possibilities are illustrated by the numerical examples in the figure. Unit A takes in more than it puts out - 10 kg/s in and 9 kg/s out. Unit B takes in less than it puts out - 10 kg/s in and 11 kg/s out. Unit C takes in and puts out the same amount - 10 kg/s in and 10 kg/s out.

Units A and B clearly situations that have never been observed in any long term flow situation. And they seem quite impossible. So, Unit C represents what can actually happen. While this is true for all time rate of flow of matter, here let's restrict ourselves to fluids.

If it existed, Unit A would be a Flood Killer. Imagine, every second you pour in 10 kg of water and only 9 kg comes out - a 10% loss per pouring. More and more water disappears without a trace! Water destroyed without any other effects!

If it existed, Unit B would be a Desert Buster. Image, every second you pour 10 kg of water in and get 11 kg out - a 10% increase per pouring. More and more water is created without any other effects.

This established the conservation of matter in flow situations. Now we will obtain a mathematical expression for his situation for fluid flow. The relationship is sets the stage for the Equation of Continuity. The Equation of Continuity relates the incoming and outgoing densities, areas and velocities . Now let's quantify it.

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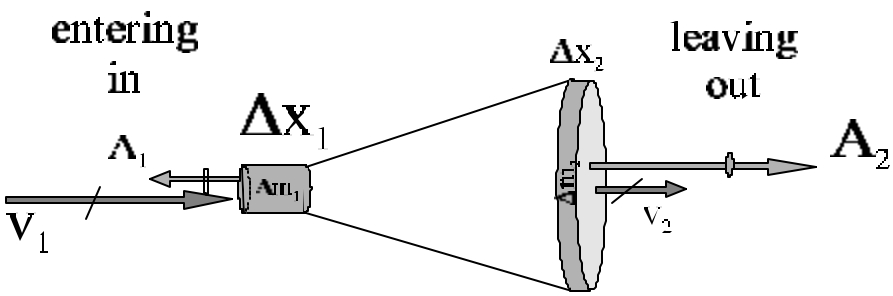
Conservation of Matter

$$\frac{\text{netmatterentering}(\Delta m_1)}{\text{perunittime}(\Delta t)} = \frac{\text{netmatterleaving}(\Delta m_2)}{\text{perunittime}(\Delta t)}$$

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$



Entering or in

$$\frac{\Delta m_1}{\Delta t} = \rho_1 \frac{\Delta Vol_1}{\Delta t}$$
$$= \rho_1 \frac{A_1 \Delta x_1}{\Delta t}$$
$$= \rho_1 A_1 \frac{\Delta x_1}{\Delta t}$$
$$= \rho_1 A_1 v_1$$

Leaving or out

$$\frac{\Delta m_2}{\Delta t} = \rho_2 \frac{\Delta Vol_2}{\Delta t}$$
$$= \rho_2 \frac{A_2 \Delta x_2}{\Delta t}$$
$$= \rho_2 A_2 \frac{\Delta x_2}{\Delta t}$$
$$= \rho_2 A_2 v_2$$

Both sides are equal to each other. This gives us the **Equation of Continuity**

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Examples:

1. In the wide deep river downstream, the water hardly moves. But, upstream where it is narrow and shallow the water runs dangerously fast!

Equation of Continuity			**r =**	**constant**
	Water along a river			
Width (m)	1000	100	20	5
Depth (m)	20	10	3	2
A (m^2)	20000	1000	60	10
v (m/s)	0.10	2.00	33.33	200.00
v (f/s)	0.3281	6.562	109.36667	656.2
v (mph)	2.24E-01	4.47E+00	7.46E+01	4.47E+02
	At the mouth of the river		white water rapids	River would dig a bigger channel

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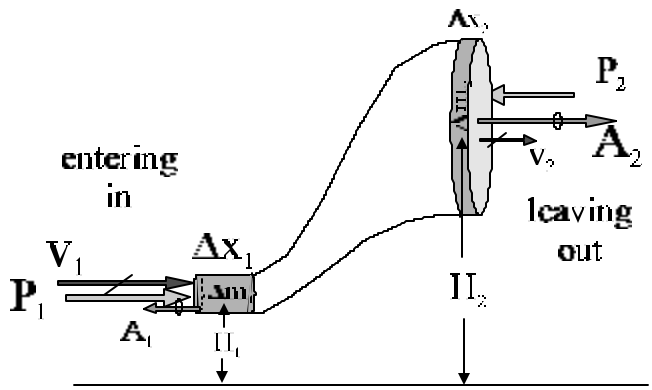
15.7.2 Bernoulli's Principle - conservation of energy



Conservation of Energy

This is a cut away section of an extended flow of a fluid in an "enclosure". The "enclosure" is part of a continuous system carrying the fluid. The boundaries of the enclosure can be a physically enclosed pipe or streamlines in an large open system like the ocean or air flow around an object. The relative sizes of the ends can be as small or as large as you like. On the left, Position 1, the fluid is being pushed into the enclosure by the pressure P_1 caused by the fluid behind it outside the enclosure as shown. On the right, Position 2, the fluid in the enclosure is pushing against the fluid in front of it as it moves out. By Newton's 3rd Law the fluid in front of the enclosure pushes back on the fluid in the enclosure. This is shown by the pressure P_2. The Work-Energy Theorem and the fact that work = F Δx, gives us

$$\frac{1}{2}m_2v_2^{\,2} + m_2gH_2 \;=\; \frac{1}{2}m_1v_1^{\,2} + m_1gH_1 + \mathrm{Work}_{1\to 2}$$

At position 1, the fluid behind does work $F_1\Delta x_1$ as it pushes the fluid into the enclosure. At position 2, the fluid in the enclosure does work $F_2\Delta x_2$ on the fluid in front of it as it pushes the fluid outside ahead of it. This is a negative work done on the fluid in the enclosure. (You can experience this when you are in a crowd trying to get into a crowded room. The people behind you are pushing you in, those in front are pushing back on you to impede your forward movement.) So,

$$\mathrm{Work}_{1\text{->}2} = F_1\Delta x_1 - F_2\Delta x_2$$

giving

$$\frac{1}{2}m_2v_2^{\,2} + m_2gH_2 \;=\; \frac{1}{2}m_1v_1^{\,2} + m_1gH_1 + F_1\,\Delta x_1 - F_2\,\Delta x_2$$

Getting all for position 1 on one side of the equation and for position 2 on the other

$$\frac{1}{2}m_2v_2^{\,2} + m_2gH + F_2\Delta x_2 \;=\; \frac{1}{2}m_1v_1^{\,2} + m_1gH_1 + F_1\Delta x_1$$

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Using $m = \rho \Delta Vol$ and $F=P\Delta x$, and noting that P_2 is opposite to the displacement Δx_2, gives

$$\frac{1}{2}\rho_2 \Delta Vol_2 v_2^{\ 2} + \rho_2 \Delta Vol_2 g H_2 + P_2 A_2 \Delta x_2 = \frac{1}{2}\rho_1 \Delta Vol_1 v_1^{\ 2} + \rho_1 \Delta Vol_1 g H_1 + P_1 A_1 \Delta x_1$$

Using $\Delta Vol = A\,\Delta x$,

$$\frac{1}{2}\rho_2 \Delta Vol_2 v_2^{\ 2} + \rho_2 \Delta Vol_2 g H_2 + P_2 \Delta Vol_2 = \frac{1}{2}\rho_1 \Delta Vol_1 v_1^{\ 2} + \rho_1 \Delta Vol_1 g H_1 + P_1 \Delta Vol_1$$

Factoring our $\rho A \Delta x$ on each side, dividing by Δt and using the definition $v = \Delta x/\Delta t$, gives

$$\left(\frac{1}{2} v_2^{\ 2} + gH_2 + \frac{P_2}{\rho_2}\right)\rho_2 A_2 v_2 = \left(\frac{1}{2} v_1^{\ 2} + gH_1 + \frac{P_1}{\rho_1}\right)\rho_1 A_1 v_1$$

From the **Continuity Equation** $\mathbf{r_1 A_1 v_1 = r_2 A_2 v_2}$, so they cancel leaving

Bernoulli's Principle

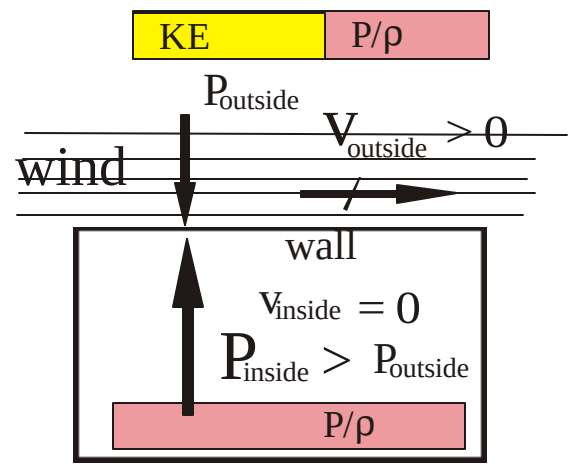
$$\frac{1}{2} v_2^{\ 2} + gH_2 + \frac{P_2}{\rho_2} = \frac{1}{2} v_1^{\ 2} + gH_1 + \frac{P_1}{\rho_1}$$

Examples:

1. In hurricanes and tornadoes the walls of building are pushed outward. The air inside has zero velocity while the outside air has a very high velocity. So the inside pressure is higher than the pressure outside - the walls get pushed out by the high inside pressure. The same is true for the roof.

$$P_{inside} - P_{outside} = \tfrac{1}{2}\rho_{air}\, v^2_{outside}$$

The energy blocks illustrate the distribution of kinetic and pressure potential energies in the two regions.



Pinside - Poutside							
v (m/s)	10	30	40	60	80	100	120
v (f/s)	32.81	98.43	131.24	196.86	262.48	328.1	393.72
v (mph)	2.24E+01	6.71E+01	8.95E+01	1.34E+02	1.79E+02	2.24E+02	2.68E+02
Pin-Pout (N/m^2)	64.5	580.5	1032	2322	4128	6450	9288
Pin-Pout (p/in^2)	9.35E-03	8.42E-02	1.50E-01	3.37E-01	5.99E-01	9.35E-01	1.35E+00
Energy/Vol (J/m^3)	64.5	580.5	1032	2322	4128	6450	9288
			Hurricane				Tornado

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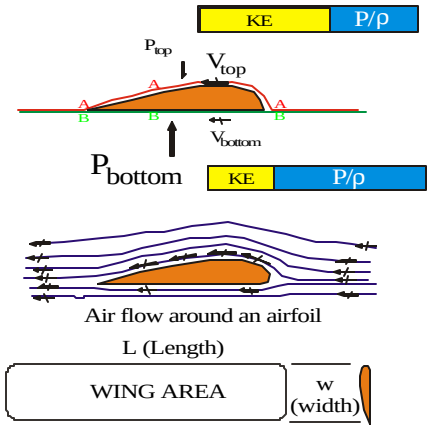
2. As an airplane wing (airfoil) cuts through the air, the air is split into two streams that meet again at the tail of the wing. One stream goes over the top of the wing, the other under the bottom. The stream going over the curved top of the airplane wing travels a longer path than that going under the flatter bottom of the wing. The effective velocities are $v_{top} = L_{top}/\Delta t$ and $v_{bottom} = L_{bottom}/\Delta t$, where Δt is the time the two streams simultaneously travel over the wing. Since $L_{top} > L_{bottom}$, then $v_{top} > v_{bottom}$. Bernoulli's Principle gives



$$P_{lift} = P_{top} - P_{bottom} = \tfrac{1}{2}\rho_{air} v^2_{bottom}[(L_{top}/L_{bottom})^2-1].$$

So, the lift of the airplane wing is $F_{lift} = P_{lift} A_{wing}$,

$$F_{lift} = \tfrac{1}{2}\rho_{air} v^2_{bottom}[(L_{top}/L_{bottom})^2-1]\, A_{wing}$$

Setting B as the airfoil geometric parameter defined as

$$B = [(L_{top}/L_{bottom})^2-1],$$

we get for P_{lift} and F_{lift}

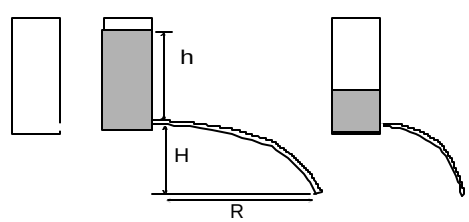
$$P_{lift} = \tfrac{1}{2}\rho_{air} v^2_{bottom} B \quad \text{and} \quad F_{lift} = \tfrac{1}{2}\rho_{air} v^2_{bottom} B\, A_{wing} \quad .$$

This model is a simplification of reality. Turbulence, The Equation of Continuity, air friction and a few other effects also play important roles. But, Bernoulli's Principle lies at the base.

	Lift Pressure of Airfoils							
		Ltop/Lbott	1.01	1.02	1.05	1.1	1.2	1.5
		B	0.0201	0.0404	0.1025	0.21	0.44	1.25
	v (m/s)	v (mph)						
	10	22.37	1.27E+01 N/m^2	2.56E+01	6.49E+01	1.33E+02	2.78E+02	7.91E+02
	30	67.11	1.14E+02 N/m^2	2.30E+02	5.84E+02	1.20E+03	2.51E+03	7.12E+03
	50	111.85	3.18E+02 N/m^2	6.39E+02	1.62E+03	3.32E+03	6.96E+03	1.98E+04
Jet Takeoff	80	178.96	8.14E+02 N/m^2	1.64E+03	4.15E+03	8.50E+03	1.78E+04	5.06E+04
Jet cruising	250	559.25	7.95E+03 N/m^2	1.60E+04	4.05E+04	8.30E+04	1.74E+05	4.94E+05

3. Watch the water stream falling from your faucet when the flow is steady. Why does it get narrower as it falls?

4. A can or bottle has a small hole near its bottom. The has water at a height h above the hole and the hole is a height H above the ground. The water stream from the hole hits the ground a horizontal distance R from the hole as shown in the picture.



   1. Show that $H=R^2/4h$ (derive it).
   2. Do it. Perform it as an experiment.
      Set H fixed, measure h and R. Do it again for a total of at least three values

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of H. Compare measured values with the above equation.

Sample Data Sheet

H (fixed)	Measured		Calculated		% Diff	
	h	R	h	R	h	R

15.7.4 Size Effect The Smaller the Body, the Smaller the Liftoff Speed.

Let v_{bottom} be the velocity v of the object. The lift force must equal the weight of the body,.

$$\text{Weight} = \rho_{body} Vol_{body} = F_{lift} = \tfrac{1}{2}\rho_{air} v^2 B A_{\text{"wing"}} ,$$

where $B = [L_{top}/L_{bottom})^2 - 1]$ is the air foil geometric parameter.

If we use L as the characteristic size of the body, then

$$Vol_{body} \quad L^3 \text{ and } \quad A_{\text{"wing"}} \sim L^2 \text{ . Hence}$$

$$\rho_{body} Vol_{body} = \tfrac{1}{2}\rho_{air} v^2 B A_{\text{"wing"}} \quad \text{becomes} \quad \rho_{body} L^3 \sim \tfrac{1}{2}\rho_{air} v^2 B \sim L^2 \quad .$$

A little algebra gives us

$$v^2 \sim 2\,(\rho_{body}/\rho_{air})\, L/B \,.$$

Hence the smaller the body, the lower the speed it needs for "take off".
$\rho_{body} = 6\text{x}10^3$ kg/m^3 6 times the density of water. And $\rho_{body}/\rho_{air} = 4.65 \text{ x } 10^3$.

Take off Size Effect

B	1.01		1.1		1.5	
Size L (m)	v (m/s)	v(mph)	v (m/s)	v(mph)	v (m/s)	v(mph)
1.00E-05	3.03E-01	6.79E-01	2.91E-01	6.51E-01	2.49E-01	5.57E-01
1.00E-04	9.60E-01	2.15E+00	9.20E-01	2.06E+00	7.87E-01	1.76E+00
1.00E-03	3.03E+00	6.79E+00	2.91E+00	6.51E+00	2.49E+00	5.57E+00
1.00E-02	9.60E+00	2.15E+01	9.20E+00	2.06E+01	7.87E+00	1.76E+01
1	9.60E+01	2.15E+02	9.20E+01	2.06E+02	7.87E+01	1.76E+02
1.00E+01	3.03E+02	6.79E+02	2.91E+02	6.51E+02	2.49E+02	5.57E+02
1.00E+02	9.60E+02	2.15E+03	9.20E+02	2.06E+03	7.87E+02	1.76E+03