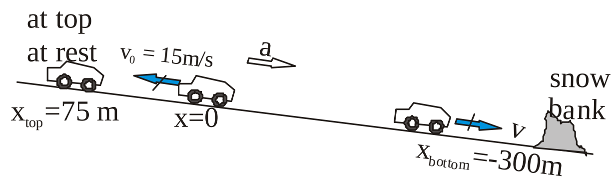
NAME____

Borough of Manhattan Community College Course ***Physics 215***

Instructor: ***Dr. Hulan E. Jack Jr.*** Date **February 25, 2003**

Quiz BC 4

A car at t=0, with an initial velocity $\mathbf{v}_0$ = 15 m/s coasts up a hill with a constant a constant downhill acceleration **a** . It coasts up the hill a displacement x_{top} = 75m and t_{top} it comes to a momentary stop. It then coasts downhill at the same acceleration until it hits a snow bank at $\mathbf{x}_{\mathbf{bottom}}$ = -300 m sudden stops after traveling a **total** time t_{total}.



a. Sketch the **v** vs t curve for the plane Label all variables. 30 points

b. State the relationship between the displacement **x** and the **v** vs t curve. 10 points

c. State the relationship between the acceleration **a** and the **v** vs t curve. 10 points

d. In terms of **initially known** variables, find the expression (equation) from which you can calculate t_{top}. **Briefly** explain what you are doing and why. 15 points

e. In terms of **initially known** variables, find the equation from which you can calculate the constant acceleration **a**. **Briefly** explain what you are doing and why. 15 points

f. Using the above, obtain an equation from which you can get t_{total} . **Briefly** explain what you are doing and why. 20 points

NAME_Me________Hulan E. Jack Jr.__My Solutions_______Feb 27, 2003_____
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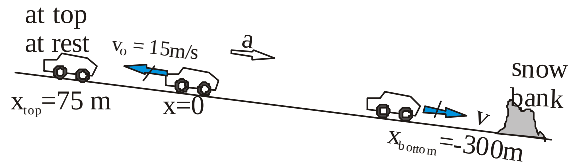
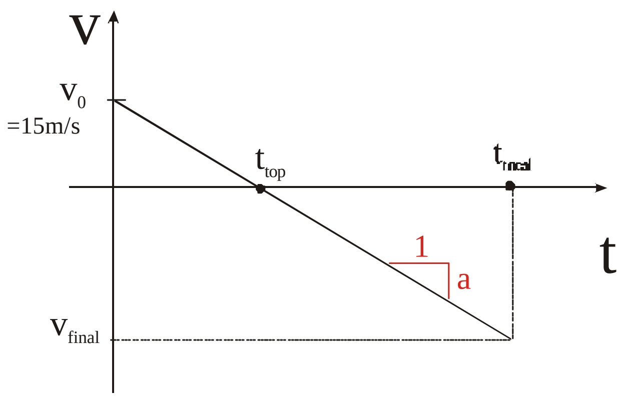
Quiz BC 4 My Solutions

A car at t=0, with an initial velocity $\mathbf{v}_0$ = 15 m/s coasts up a hill with a constant a constant downhill acceleration **a** . It coasts up the hill a displacement x_{top} = 75m and t_{top} it comes to a momentary stop. It then coasts downhill at the same acceleration until it hits a snow bank at $\mathbf{x}_{\mathbf{bottom}}$ = -300 m sudden stops after traveling a **total** time t_{total}.



a. Sketch the **v** vs t curve for the plane Label all variables. 30 points



b. State the relationship between the displacement **x** and the **v** vs t curve. 10 points

x = the area under the v vs t curve.

c. State the relationship between the acceleration **a** and the **v** vs t curve. 10 points

a=slope of v vs t curve.

d. In terms of **initially known** variables, find the expression (equation) from which you can calculate t_{top}. **Briefly** explain what you are doing and why. 15 points

Note: All answers must be in terms of the **initially known** variables.

So, the relationship must contains t_{top} as the only unknown variable and any of the above initially known variables.

$$\Delta x = x_{top} = \text{area under v vs t curve t=0 to } t_{top} = \tfrac{1}{2} v_0 t_{top} .$$

This formula satisfies that criteria.

e. In terms of **initially known** variables, find the equation from which you can calculate the constant acceleration **a**. **Briefly** explain what you are doing and why. 15 points

$\mathbf{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_{top} - v_0}{t_{top}} = \dfrac{0 - v_0}{t_{top}} = -\dfrac{v_0}{t_{top}}$ has2 initiallyunkownvariables. So, it won'tdo.

But, itdoesgive $t_{top} = -\dfrac{v_0}{a}$. Substitutingthisinto $x_{top} = \tfrac{1}{2} v_0 t_{top}$ ind,above, yields

$x_{top} = \tfrac{1}{2} v_0 \left(-\dfrac{v_0}{a}\right) = -\dfrac{v_0^2}{2a}$. Now **a** istheonlyinitiallyunknownvariable.

f. Using the above, obtain an equation from which you can get t_{total} . **Briefly** explain what you are doing and why. 20 points

The simplest solution is the fact that the area under the triangle is the triangle from t_{top} to t_{total} the displacement $x(t_{total}) - x(t_{top})$ = $x_{boottom} - x_{top}$. This gives

$$x_{bottom} - x_{top} = \tfrac{1}{2} \text{ base x height} = \tfrac{1}{2} (t_{total} - t_{top} \text{ x } (a(t_{total} - t_{top}) = \tfrac{1}{2} a(t_{total} - t_{top})^2 .$$

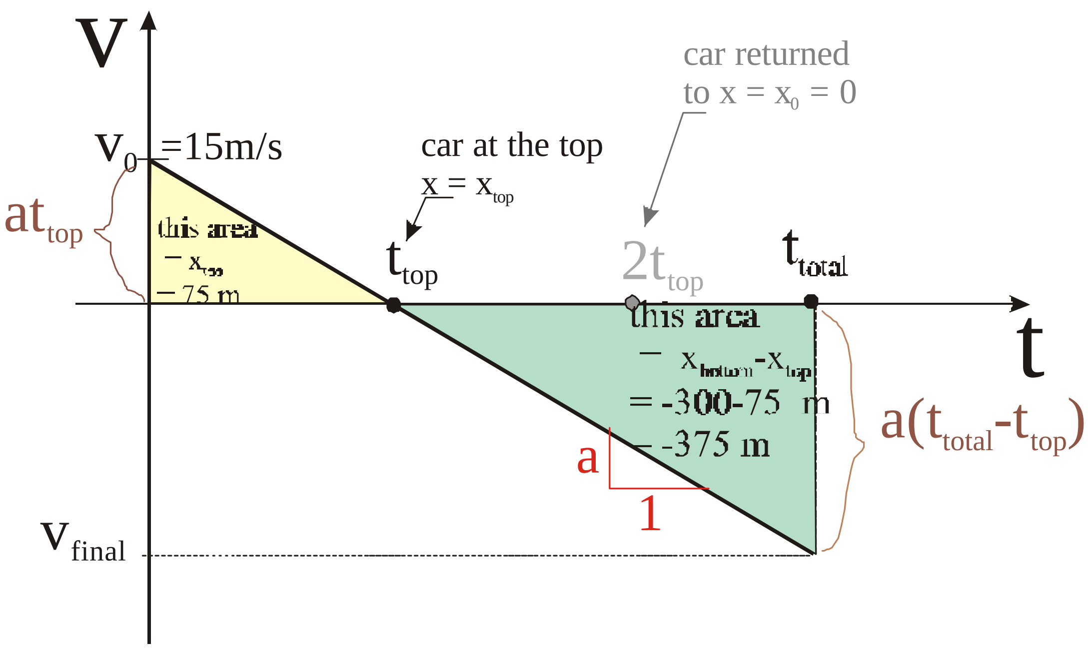
This will yield t_{total} in terms of the above.

Alternately, if you know calculus, the total displacement from t=0 to $t = t_{total}$ is then

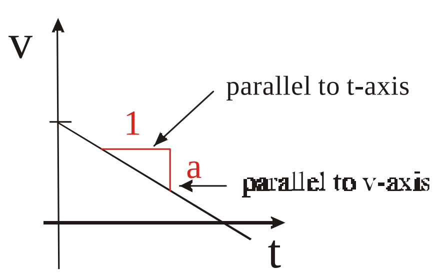
$$x(t) - x(0) = \int_o^t v(t)dt = \int_o^t (v_0 + at)dt = \int_o^t v_0 dt + \int_0^t atdt = v_0 t + \tfrac{1}{2} at^2$$

Which gives $x(t_{total}) - 0 = x_{bottom} = v_0 t_{total} + \tfrac{1}{2} at_{total}^2$. This quadraticint$_{total}$.

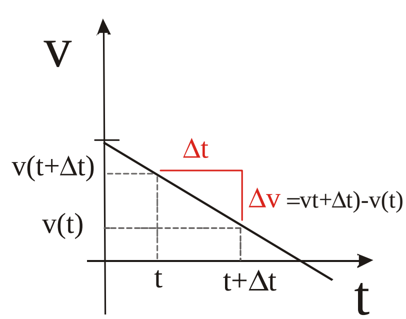
A More Complete **v vs t** Diagram



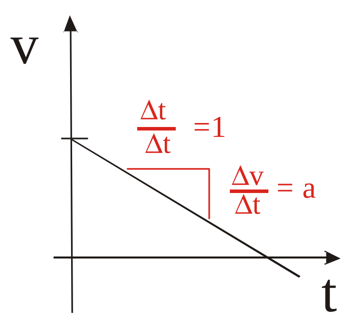
Note that the time t_{top} the car is at x_{top}. After time t_{top} the car spends another interval of t_{top} rolling back to position x_0 =0. It gets there at time $2t_{top}$. Then from $t=2t_{top}$ to t_{total}, it moves to 300 m down hill to x_{bottom} = -300.



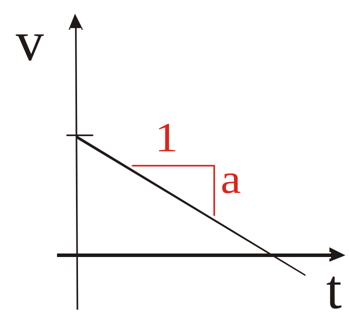
How this comes about



Marking off Δx and Δt



Divide both by Δt



Use the final values

Let MATHCAD do the math

Initial Conditions:

$a := -1.5\,\dfrac{m}{sec^2}$ $\qquad v0 := 15\,\dfrac{m}{sec}$ $\qquad x0 := 0m$

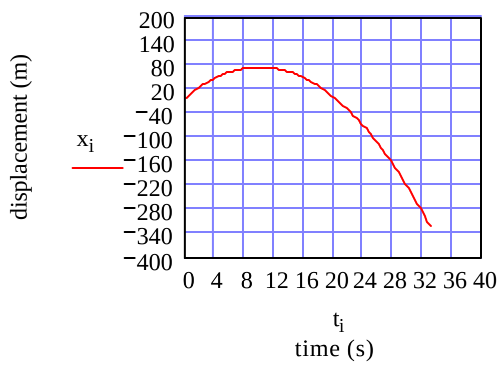
Equations of Motion:

$i := 0..99$ $\qquad t_i := \dfrac{i}{3}sec$

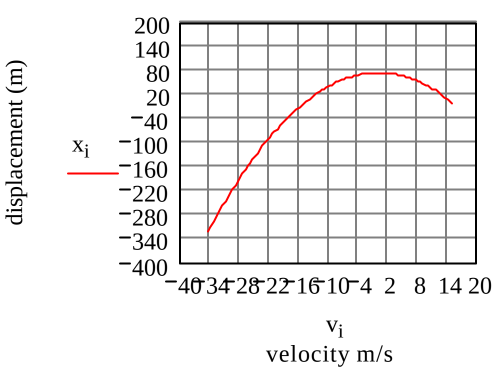
Intervals of 1/3 sec are used for the computation.

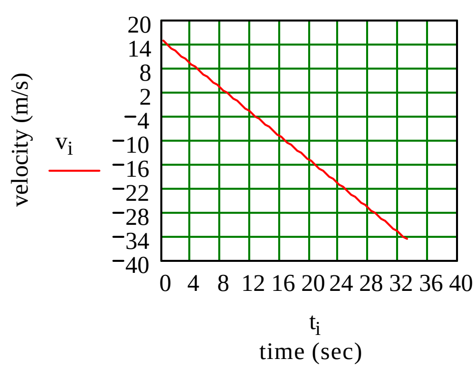
$v_i := v0 + a \cdot t_i$ $\qquad x_i := v0 \cdot t_i + 0.5 \cdot a \cdot (t_i)^2$

$t_i =$ (s)	$v_i =$ (m/s)	$x_i =$ (m)
0	15	0
0.333	14.5	4.917
0.667	14	9.667
1	13.5	14.25
1.333	13	18.667
1.667	12.5	22.917
2	12	27
2.333	11.5	30.917
2.667		34.667
		38.25



These tables can be extended to show the full range of values





The graph shows how the displacement and velocity are related.

Now let's solve for the time the whole trips takes when xf = -300 m. This value can be changed at will.

$xf := -300$ $x01 := 0$ $a1 := -1.5$ $v01 := 15$

This method of getting roots does not allow units. So, values must be re-entered with new names.

$$f(t1) := xf - x01 - v01 \cdot t1 - 0.5 \cdot a1 \cdot t1^2$$

$$v := f(t1) \text{ coeffs}, t1 \rightarrow \begin{pmatrix} -300 \\ -15 \\ .75 \end{pmatrix}$$

This converts the coefficients of f(t1) into a vector. Unfortunately, **every element of a vector must have the same units**. That is why this method does not allow units.

$$s := \text{polyroots}(v)$$

$$s^T = (-12.361 \quad 32.361)$$

The -12.361 is meaningless because that is before the problem started. So, th correct answer is 32.361 sec.

NOTES:
1. The work done here would take a very experienced computationists 5 to 10 hours using a calculator to create the tables of values and to draw the graphs. This does not include the time for the annotating notes.
2. Setting up this worksheet takes less than a half hour for someone experienced with MathCad, and over two hours for someone inexperienced. But, once setup any input value can be changed at will and in an "instant" everything is redone to reflect the change.