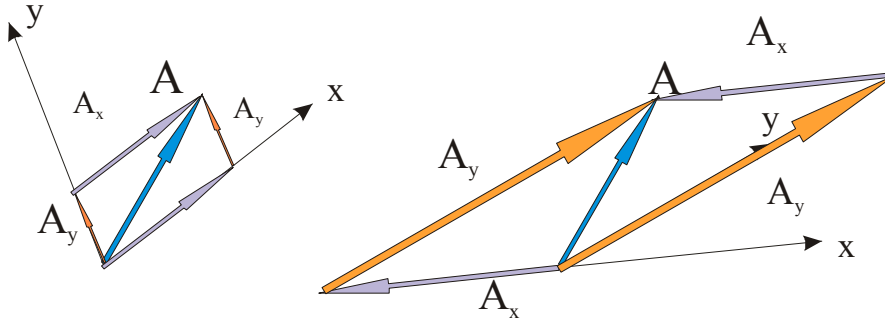


Boot Camp Final

1. Sketch the A_x and A_y components in each picture. Do it on the pictures.

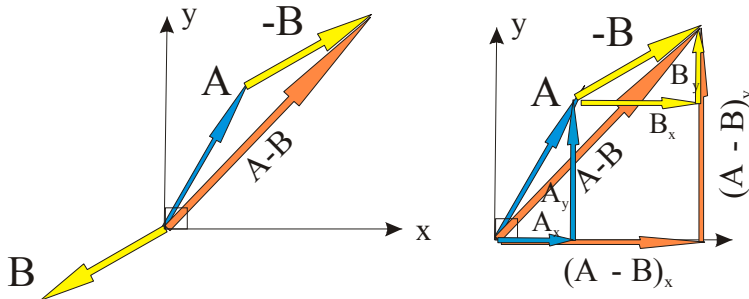


NOT ASKED FOR FOR THE EXAM

For components, A_x is \parallel to x axis, A_y is \parallel to y axis, and $A = A_x + A_y$.

2. Sketch the vector $A - B$, then the components $A_x, A_y, (-B)_x, (-B)_y, (A-B)_x, (A-B)_y$. You can do them on the picture.

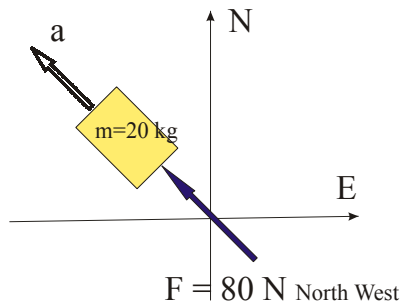
3 points
2 points each set



The two are done separately for clarity.

In trying to understand, I push myself to the limits of frustration.
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3. A net force $F = 80 \text{ N}$ North West acts on a body of mass $m = 20 \text{ kg}$.
 What is the body's acceleration, a ? Include units. 5 points
 Sketch a picture of the situation. 5 points



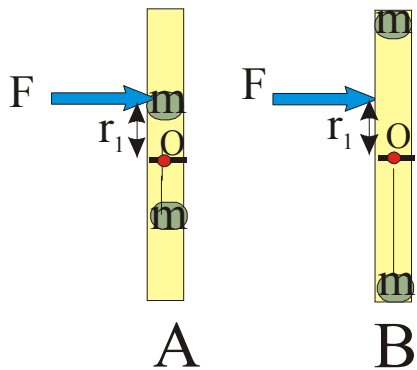
$$F = ma.$$

$$\text{So, } a = F/m$$

$$a = 80 \text{ N North West} / 20 \text{ kg} = 4 \text{ m/s}^2 \text{ North West.}$$

The acceleration is in the same direction as the force.

4. You have a massless stick and two blobs both having a mass m . The center of stick is marked by the line with the dot. Two placements of the blobs are shown in the figures A and B. The blobs are the same distance from the center. In each, a force F is applied at a distance r_1 from the center of the stick.
 Briefly describe what happens in each situation. 3 points



Answer below.

Definition

2 points

$$\text{torque } \tau_o = r \times F = rF_{\perp}$$

Moment of

$$\text{Inertia } I_o = \sum m_i r_{io}^2$$

Details for this situation

3 points

$$\tau_o = r_1 F$$

$$I_o = m r_1^2 + m r_1^2 = 2m r_1^2 .$$

In each situation the stick accelerates both linearly and rotationally. By Newton's 2nd Law of linear motion, $F=ma$, the linear acceleration will be the same in both situations.

Relation between them. 2 points

But rotation is different. Newton's 2nd Law for rotation is $\sum \tau_o = I_o \alpha$, where α is the angular acceleration. As r_1 gets larger, I_o increases (by square) the angular acceleration decreases.

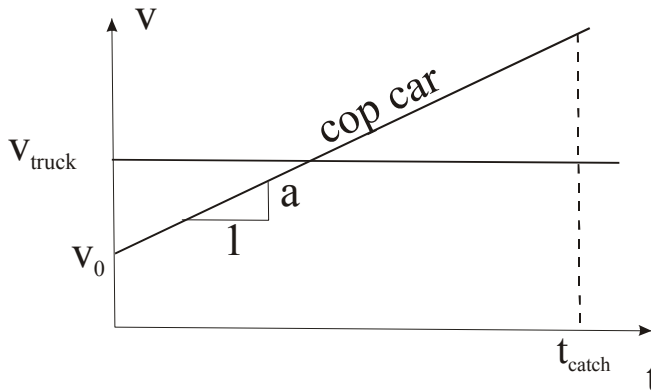
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5. A cop car is traveling along at a velocity $v_0 = 20$ m/s when a truck passes it traveling a velocity $v_{\text{truck}} = 40$ m/s. The car immediately accelerates at the rate $a = 3$ m/s². At time t_{catch} the cop car catches up with the speeding truck. This is shown on the v vs t curve. At t_{catch} what must be true? How would you use it to find the value of t_{catch} ? Explain.



At catchup the two vehicles have traveled the same displacement. Hence the area under their v vs t curves are the same. The areas depend on t_{catch} . Some algebra gives t_{catch} .

NOT REQUIRED FOR THE EXAM

$$x_{\text{truck}} = x_{\text{car}}$$

$$v_{\text{truck}} t_{\text{catch}} = v_0 t_{\text{catch}} + \frac{1}{2} a t_{\text{catch}}^2$$

This yields $t_{\text{catch}} = 2 (v_{\text{truck}} - v_0) / a$

6. A person of mass $m = 80$ kg. sits in the seat of an airplane. The airplane is momentarily at the bottom of a dive traveling a vertical circle of radius $R = 2000$ m. Its velocity $v = 200$ m/s. Sketch the Free Body Diagram of the person (use the figure), state principles and fill in the details.

FBD of person	State Principles	Fill in the Details
$a_c = v^2 / R$ 	4pts $+\uparrow \Sigma F = ma$ $a = a_c = v^2 / R$	3 pts $N - mg = mv^2 / R$

Note: Always put forces where they actually act. The Normal, N , acts on the bottom of the person by the seat pushing them up, NOT ON THE TOP AS IF THEY ARE HANGING!

Analysis (Not required for exam)

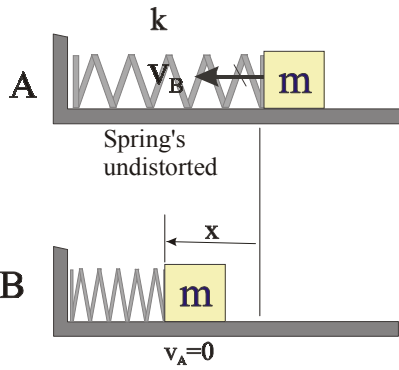
The Normal, N , is force that the seat pushes up on the person. It is the “effective” of the person.

$$N = mg + mv^2/R = mg(1 + v^2/(gR)) \quad \text{factoring out } mg$$

$$= W(1 + v^2/(gR)) \quad \text{Since } W = mg$$

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7. A block of mass $m = 10 \text{ kg}$ travels along a horizontal rough plane which causes a kinetic friction force \mathcal{F} on the block. The block has a velocity $v_0 = 20 \text{ m/s}$ when it makes contact with a free spring of stiffness $k = 10000 \text{ N/m}$. The block comes to rest after compressing the spring by $x = 0.1 \text{ m}$. Find the value of the friction force \mathcal{F} .



Principles

Details (Symbols Only)

Conservation of Energy

$$PE_A + KE_A = PE_B + KE_B + \text{Work}_{\text{Other}}$$

DETAILS

$$\begin{array}{ccccccc}
 0 & + & \frac{1}{2} m v_0^2 & = & \frac{1}{2} k x^2 & + & 0 + \mathcal{F} x \\
 \text{Spring undeformed} & & & & \text{Spring compressed} & & \text{stopped}
 \end{array}$$

NOT REQUIRED FOR THE EXAM

After some algebra

$$\mathcal{F} = \frac{1}{2} (m v_0^2 - k x^2) / x$$

8. Derive that for a distance R from the center of a massive body of mass M that the gravitational intensity $g = GM/R^2$. R is R_{Surface} , or larger; $R > R_{\text{Surface}}$.

Newton's law of Gravitation is $F_G = - GMm/R^2$

Weight, W , is the force due to gravity.

So, $W = F_G = GmM/R^2$ when $R > R_s$, where R_s is the radius of the massive body.

Newton's 2nd Law of Motion (linear) is $F=ma$. For weight acting acceleration is due to the acceleration due to gravity. Hence $a = g$.

Combining these yields $W = mg = GmM/R^2$. Cancelling m gives $g = GM/R^2$ for $R > R_s$.

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9. The equation of motion of a physical system is found to be

$$(I_0 + ML^2) \alpha = - MgL\theta .$$

It is released from rest at an angle θ_0 .

- A. What is the frequency of this motion. Explain, show your arguments.
6 points

Simple Harmonic Motion is characterized by acceleration = $-(2\pi f)^2$ displacement, giving
 $f = (1/2\pi) \text{sqrt}(\text{acceleration}/\text{displacement})$.

Here the acceleration is the angular acceleration α and the displacement , the angular displacement θ . Hence ,

$$f = (1/2\pi) \text{MgL}/(I_0 + ML^2)$$

- B. In terms of θ_0 , I_0 , L and M , write the equation describing motion of this system. Explain.

4 points

Simple Harmonic Motion is described by the equation Displacement = Amplitude $\sin(2\pi t)$.

Here the displacement is the angular displacement θ , the amplitude θ_0 . So,

$$\theta = \theta_0 \sin(\text{MgL}/(I_0 + ML^2) t)$$

10. What is stress? Give its definition. 3points

By definition, Stress $\sigma = \text{Force}/ \text{Area} = F/A$

What is the response to stress? Give its name and definition. 3 points

The response to stress is strain, ϵ , defined as $\epsilon = \Delta L /L_0$

How are they related? 1 point

The two are related by the elastic modulus, M, of the materials by

$$\sigma = M \epsilon$$

Name the three types of stress. 3 points

the three types of stress are axial, shear, and pressure.

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